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The Question of Drawing a Design Scheme of a Wagon with a Non-Symmetrically Located Freight

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Abstract. Violations of the stability of a wagon with an asymmetrically positioned freight can lead to a threat to the safety of train traffic, the safety of freight on the route and transport facilities. The design diagram of a wagon with an asymmetrically located freight drawn up in the article and the equations obtained in an analytical way, oscillations of the sprung mass of a wagon with an asymmetric arrangement of the freight, characterizing the longitudinal and transverse displacement of the center of mass of the freight, can be used for mathematical modeling of forced vibrations and impact on the track of a wagon with asymmetrically located freight in various operating conditions.

Keywords: wagon, symmetrical system, asymmetric system, oscillations, freight.

1. Introduction

Today, in wagon bogies, as a rule, friction dampers are used, in which vibration resistance is created due to dry friction between the parts of the friction damper. It is assumed in the article that the friction dampers are designed so that the normal pressure between the rubbing surfaces is proportional to the amount of compression of the elastic elements of the spring suspension [1]. To determine the amplitude-frequency response, we first set certain values of the damper resistances and varied the stiffness of the spring set, which made it possible to establish the effect of changes in stiffness in a fairly wide range on the character and magnitude of the amplitude-frequency response. It was found that the optimal conditions for the interaction of a wagon and a railway track will be provided if the range of vibration frequency at which the modulus of the amplitude-frequency characteristic reaches the highest value (for each of the stiffnesses) does not coincide with the frequencies of the maximum value of the spectral density of the impact of irregularities in the path. Let us consider the main features of the design model of a wagon with an asymmetric placement of freight [2]. It is known that when the freight is symmetrically located in the wagon, the axes of symmetry of the body coincide with the main axes of inertia of the body and the freight, and as with an asymmetric arrangement of the freight, the axes of symmetry of the body and the freight do not coincide [3]. If for an asymmetric system it is assumed that the coordinate axes coincide with the symmetry axes of the body, then the inertial properties of solids can be expressed through the moments of inertia about three mutually perpendicular axes and the centrifugal moments of inertia corresponding to these axes.

2. Materials and Methods

In the case when two centrifugal moments of inertia with one axis index are the main axis of inertia and the moment of inertia relative to it is the main moment of inertia, the body of a wagon is a symmetrical system [4]. The freight is assumed to be symmetrical: its axes are parallel to the axes of symmetry of the body, and the center of gravity is displaced along the carriage by the value x, across the carriage by the value y, and vertically by the value z (see Figure 1). As generalized coordinates, it is possible to take displacements along the axes of symmetry and turns of the body with freight relative to the axes of symmetry of the body. In this case, the expression of kinetic energy has a complex form and there are dynamic links in the system, but one can act differently and take displacements along the axes of inertia and rotations of the body with freight relative to these axes as generalized coordinates. In this case, the expression for the kinetic energy is simplified, and for the potential energy, it becomes more complicated. The general problems of determining the principal axes of symmetry of a rigid body at a given point, as well as the main moments, are given in [2]. To find the main axes of inertia, it is necessary to determine the angles α, β, γ between the main and original coordinate axes, which are related by the relation:

$$\alpha^2 + \beta^2 + \gamma^2 = 1 \tag{1}$$

and report the extreme value to a homogeneous quadratic form:

$$I = I_x\alpha^2 + I_y\beta^2 + I_z\gamma^2 - 2I_{xy}\alpha\beta - 2I_{zx}\gamma\alpha - 2I_{zy}\beta\gamma \tag{2}$$

where I_x, I_y, I_z – is the moments of inertia with respect to the coordinate axes; I_{xy}, I_{zx}, I_{zy} – is the centrifugal moments of inertia.

To determine the main moments of inertia, it is necessary to consider the system of equations:

$$\left.\begin{aligned} (I_x - I)\alpha - I_{xy}\beta - I_{zx}\gamma = 0 \\ -I_{xy}\alpha + (I_y - I)\beta - I_{yz}\gamma = 0 \\ -I_{zx}\alpha - I_{yz}\beta + (I_z - I)\gamma = 0 \end{aligned}\right\} \tag{3}$$

That is, a system of equations has a nonzero solution only if its determinant is zero.

We obtain an algebraic equation of the third degree with respect to I:

$$I^3 - \sigma_1 I^2 - \sigma_2 I - \sigma_3 = 0 \tag{4}$$

α_i, β_i, γ_i denote the angles between the axes of coordinates O_x, O_y, O_z, and the main axes, which correspond to the moment of inertia I_i. To determine α_i, β_i, γ_i according to [5], we obtain the equations:

$$\left.\begin{aligned} (I_x - I_i)\alpha_i - I_{xy}\beta_i - I_{zx}\gamma_i = 0 \\ -I_{xy}\alpha_i + (I_y - I_i)\beta_i - I_{yz}\gamma_i = 0 \\ -I_{zx}\alpha_i - I_{yz}\beta_i + (I_z - I_i)\gamma_i = 0 \end{aligned}\right\} \tag{5}$$

The roots of this equation are the main moments of inertia. In equation (5), the invariants of the tensor of inertia:

$$\sigma_1 = I_x + I_y + I_z$$

$$\sigma_2 = I_x I_y + I_y I_z + I_z I_x - I_{xy}^2 - I_{yz}^2 - I_{zx}^2$$

$$\sigma_3 = \begin{vmatrix} I_x & I_{xy} & I_{xz} \\ I_{yx} & I_y & I_{yz} \\ I_{zx} & I_{zy} & I_z \end{vmatrix}$$

To determine the three unknown direction cosines, Eq. (1) and two equations from Eq. (5) are used. The direction cosines of the angles α, β and γ are determined from Eq. (5) by setting sequentially by the values $i = 1, 2, 3$.

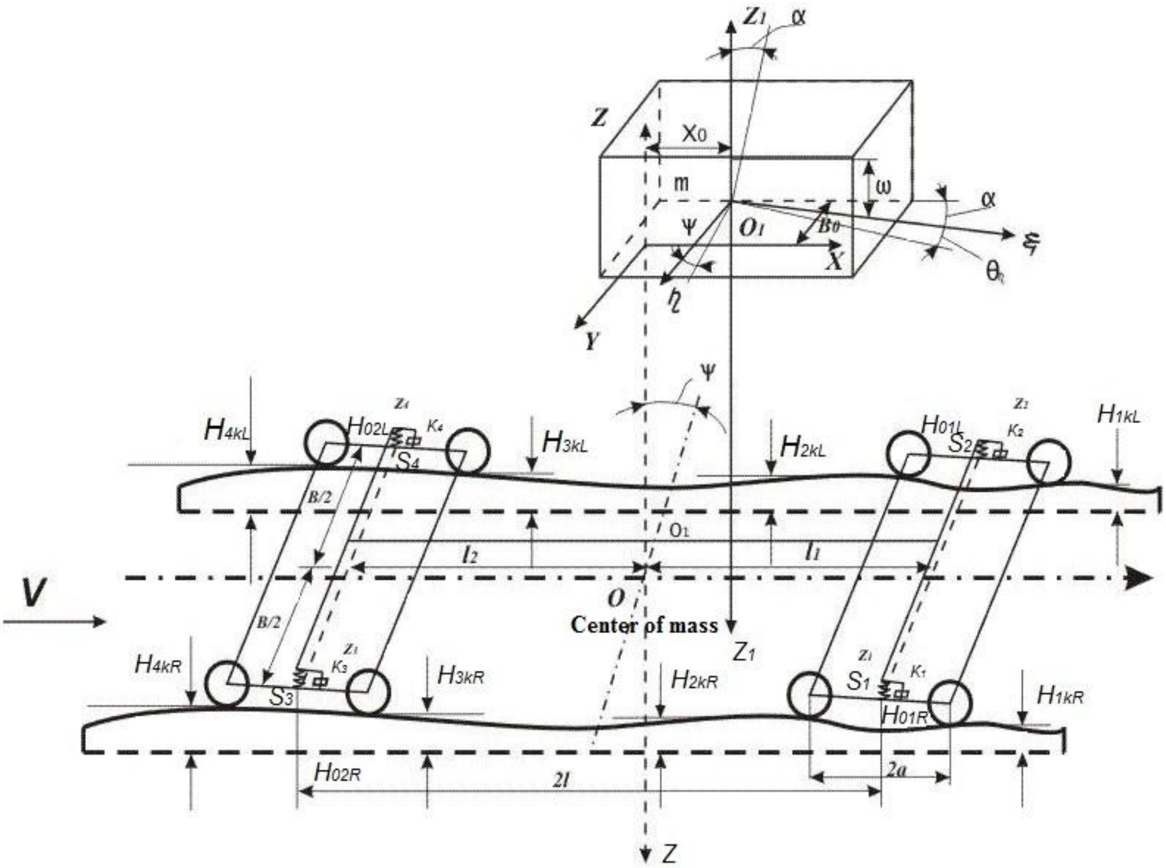


Figure 1. Design model of a wagon with asymmetrically positioned freight.

The calculations show that $\cos\beta_1$, $\cos\gamma_1$, $\cos\alpha_2$, $\cos\gamma_3$, $\cos\alpha_3$, $\cos\beta_3$ are so small with longitudinal and transverse displacements of the center of mass of the freight, even respectively up to 4.0 m from the transverse axis of symmetry of the wagon and up to 0.8 m from the longitudinal axis of symmetry of the wagon, that they can be taken almost equal to zero, and the angles $\cos\alpha_1$, $\cos\beta_2$, and $\cos\gamma_3$ must be taken into account when drawing up differential equations, since their values are significantly significant compared to $\cos\beta_1$, $\cos\gamma_1$, $\cos\alpha_2$, $\cos\gamma_3$, $\cos\alpha_3$, $\cos\beta_3$. When considering the oscillations of bouncing, galloping, and side rolling of a loaded wagon with an asymmetrically placed freight, only $\cos\alpha_1$ and $\cos\beta_2$ are taken into account. The values of the angles α_1, β_2 and γ_3 obtained as a result of calculations are given in Table 1, from which it can be seen that these angles must be taken into account only for longitudinal displacements of the center of mass of the freight over 2.0 m from the transverse axis of symmetry of the wagon, and with the transverse displacement of the center of mass of the freight more than 0.4 m from the longitudinal axis of symmetry of the wagon, as well as with a freight mass of more than 30-40 tons. With smaller values of the displacement of the center of mass of the freight and its smaller mass, the coordinate axes can be chosen to coincide with the axes of symmetry of the wagon without compromising the accuracy of calculations and assessing the qualitative characteristics of the dynamic processes of the wagon [6].

For goods weighing 20, 30, 40, 50, and 60 tons, the values of the angles α_1, β_2, and γ_3 are in the range of values given in Table 1, and vary approximately proportionally to the values of the axial and centrifugal moments of inertia of the transported goods, with a decrease in which the values of the angles α_1, β_2, and γ_3 decrease. With an increase in the lateral displacement of the center of mass of the freight, the angles β_2, and γ_3, increase, and with an increase in the longitudinal displacement of the center of mass of the freight, the angle α_1 generally increases [7]. The increase in the angles α_1, β_2 and γ_3 is especially affected by the longitudinal displacement of the center of mass of the freight over 3.0 m from the transverse axis of symmetry of the wagon and the lateral displacement of the center of mass of the freight over 0.4 m from the longitudinal axis of symmetry of the wagon.

Table 1. Angles between the coordinate axes and the main central axes of inertia I_1, I_2, I_3.

Displacement of the center of mass of the freight, m		Freight weight, t	Angle values		
			α_1 for I_1	β_2 for I_2	γ_3 for I_3
x_0 = 0,5 ;	b_0 = 0,1	10	$0^0 13'$	$0^0 30'$	$0^0 18'$
x_0 = 0,5 ;	b_0 = 0,8		$0^0 14'$	$2^0 08'$	$1^0 02'$
x_0 = 1,0 ;	b_0 = 0,1		$2^0 23'$	$0^0 39'$	$0^0 24'$
x_0 = 1,0 ;	b_0 = 0,8		$2^0 26'$	$3^0 02'$	$2^0 58'$
x_0 = 2,0 ;	b_0 = 0,1		$2^0 49'$	$0^0 52'$	$0^0 29'$
x_0 = 2,0 ;	b_0 = 0,8		$2^0 50'$	$4^0 20'$	$3^0 53'$
x_0 = 3,0 ;	b_0 = 0,1		$4^0 12'$	$1^0 09'$	$0^0 38'$
x_0 = 3,0 ;	b_0 = 0,8		$4^0 20'$	$5^0 37'$	$4^0 48'$
x_0 = 4,0 ;	b_0 = 0,1		$5^0 20'$	$1^0 12'$	$0^0 55'$
x_0 = 4,0 ;	b_0 = 0,8		$5^0 41'$	$7^0 02'$	$5^0 38'$
x_0 = 0,5 ;	b_0 = 0,1	70	$0^0 41'$	$1^0 08'$	$0^0 29'$
x_0 = 0,5 ;	b_0 = 0,8		$0^0 39'$	$5^0 47'$	$2^0 24'$
x_0 = 1,0 ;	b_0 = 0,1		$7^0 38'$	$1^0 30'$	$1^0 20'$
x_0 = 1,0 ;	b_0 = 0,8		$7^0 48'$	$8^0 58'$	$7^0 50'$
x_0 = 2,0 ;	b_0 = 0,1		$7^0 55'$	$1^0 56'$	$4^0 42'$
x_0 = 2,0 ;	b_0 = 0,8		$7^0 53'$	$11^0 26'$	$9^0 38'$
x_0 = 3,0 ;	b_0 = 0,1		$11^0 20'$	$1^0 50'$	$6^0 16'$
x_0 = 3,0 ;	b_0 = 0,8		$11^0 36'$	$13^0 55'$	$11^0 17'$
x_0 = 4,0 ;	b_0 = 0,1		$15^0 18'$	$9^0 31'$	$6^0 30'$
x_0 = 4,0 ;	b_0 = 0,8		$15^0 28'$	$15^0 38'$	$11^0 28'$

The relationship between the coordinates ζ, η, θ and β, y, z of the same point in relation to two different systems of coordinate axes can be determined. Figure 1 shows various coordinate axes and a point m. The radius of the vector of the point m relative to the axes O_{xyz} is equal to the sum of the radius of the point m relative to the axes $A_{\zeta,\ \eta,\ \theta}$ and the vector r_A of point A. Introducing the unit vectors of the axes, we obtain the expression:

$$xx_0 + yy_0 + zz_0 = z_A z_0 + y_A y_0 + x_A x_0 + \bar{\zeta}\zeta_0 + \bar{\eta}\eta_0 + \bar{\theta}\theta_0$$

To facilitate the mathematical notation, according to the methodology [4], the following Table of 2 direction cosines of the angles between the coordinate axes x, y, z, and ζ, η, θ is introduced:

Table 2. Cosine values of angles between coordinate axes.

Coordinate axes	A_ξ	A_η	A_θ
O_z	a_{11}	a_{12}	a_{13}
O_y	a_{21}	a_{22}	a_{23}
O_z	a_{31}	a_{32}	a_{33}

As a result, the following relationship is obtained between the coordinate axes x, y, z, and ζ, η, θ:

$$\left.\begin{aligned} x &= x_A + a_{11}\zeta + a_{12}\eta + a_{13}\theta \\ y &= y_A + a_{21}\zeta + a_{22}\eta + a_{23}\theta \\ z &= z_A + a_{31}\xi + a_{32}\eta + a_{33}\theta \end{aligned}\right\} \quad (6)$$

These dependencies allow us to determine the distance to the points of support of the wagon frame on the spring sets. Since for the selected design scheme of a wagon with an asymmetrically located freight, the wagging of the wagon frame is not considered in this article, the third term of the equations of the relationship between the coordinate axes x, y,, z and ζ, η, θ is taken to be zero.

In the process of oscillations of the sprung mass of the wagon with the asymmetric placement of the freight, its position is determined by three generalized coordinates: ξ – is the bouncing, measured from the main axis of inertia ζ; θ_η – is the angular displacement (galloping) relative to the main central axis of inertia η; ψ_ζ – is the angular displacement (side rolling) relative to the main central axis ζ.

On the design diagram (see Figure 1), the following conventions are adopted:

H_{1kR}, H_{2kR}, H_{3kR}, H_{4kR} - is the accidental displacement of the right wheels in the direction of travel under the influence of random irregularities of the right rail line;

H_{1kL}, H_{2kL}, H_{3kL}, H_{4kL} - is the displacement of the left wheels in the direction of movement under the influence of random irregularities of the left rail line;

H_{01R}, H_{02R}, H_{01L}, H_{02L} - is the displacement of the lower supporting surfaces of the spring suspension elastic elements, respectively, of two right and two left spring sets of bogies:

$$H_{01R} = \frac{H_{1kR} + H_{2kR}}{2}; \quad H_{02R} = \frac{H_{3kR} + H_{4kR}}{2}; \quad H_{01L} = \frac{H_{1kL} + H_{2kL}}{2}; \quad H_{02L} = \frac{H_{3kL} + H_{4kL}}{2}$$

where $H_{ikR} = H_1(t),\ H_{ikL} = H_2(t)$ – is the respectively under the right and left wheels; S_1, S_2, S_3, S_4 – is the spring kits stiffness $(S_1 = S_2 = S_3 = S_4 = S_i)$; k_1, k_2, k_3, k_2 - is the equivalent resistance coefficients of frictional vibration dampers $(k_1 = k_2 = k_3 = k_4 = k_i)$; z_1, z_2, z_3, z_4 - is the displacement of the upper supporting surfaces of the elastic elements of the spring sets.

With a longitudinal displacement of the center of mass of the freight:

$$z_1 = \xi + \left[(l - x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\theta_\eta + \left[(l - x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\psi_\zeta$$

$$z_2 = \xi + \left[(l - x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\theta_\eta - \left[(l - x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\psi_\zeta$$

$$z_3 = \xi - \left[(l + x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\theta_\eta + \left[(l - x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\psi_\zeta$$

$$z_4 = \xi - \left[(l + x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\theta_\eta - \left[(l + x_0)\cos\alpha + \frac{B}{2}\cos\beta\right]\psi_\zeta$$

With a lateral displacement of the center of mass of the freight:

$$z_1 = \xi + \left[l\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\theta_\eta + \left[l\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\psi_\zeta$$

$$z_2 = \xi + \left[l\cos\alpha + (\frac{B}{2} - b_0)\cos\beta\right]\theta_\eta - \left[l\cos\alpha + (\frac{B}{2} - b_0)\cos\beta\right]\psi_\zeta$$

$$z_3 = \xi - \left[l\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\theta_\eta + \left[l\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\psi_\zeta$$

$$z_4 = \xi - \left[l\cos\alpha + (\frac{B}{2} - b_0)\cos\beta\right]\theta_\eta - \left[l\cos\alpha + (\frac{B}{2} - b_0)\cos\beta\right]\psi_\zeta$$

With simultaneous longitudinal and transverse displacement of the center of mass of the freight:

$$z_1 = \xi + \left[(l - x_0)\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\theta_\eta + \left[(l - x_0)\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\psi_\zeta$$
$$z_2 = \xi + \left[(l - x_0)\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\theta_\eta - \left[(l - x_0)\cos\alpha + (\frac{B}{2} - b_0)\cos\beta\right]\psi_\zeta$$
$$z_3 = \xi - \left[(l + x_0)\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\theta_\eta + \left[(l - x_0)\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\psi_\zeta$$
$$z_4 = \xi - \left[(l + x_0)\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\theta_\eta - \left[(l + x_0)\cos\alpha + (\frac{B}{2} + b_0)\cos\beta\right]\psi_\zeta$$

In all the above equations, ξ- is the displacement of the center of mass of the wagon with the freight.

Due to deformations of the spring sets of the front and rear bogies, the following forces will be transmitted to the wagon frame:

$$P_1 = S_i(H_{01R} - z_1) + S_i f_1 \qquad P_2 = S_i(H_{01L} - z_2) + S_i f_2$$
$$P_3 = S_i(H_{02R} - z_3) + S_i f_3 \qquad P_4 = S_i(H_{02L} - z_4) + S_i f_4$$

where f_1, f_2, f_3, f_4 – is the static deflections of spring sets, which are different due to the displacement of the center of mass of the freight.

Static deflection taking into account the displacement of the center of mass of the freight is determined by the formula:

$$f_{cm} = \frac{1}{S_i \cdot n^m_\kappa}\left[Q_{гр}(0{,}5 \pm \frac{x_0}{l})(1 \pm \frac{b_0}{h})\right] + f_\kappa$$

where S_i – spring kits stiffness; $n^m{}_\kappa$ – number of spring kits in the bogie; $Q_{гр}$ – weight of the transported freight; x_0 – longitudinal displacement of the center of mass of the freight from the transverse axis of symmetry of the wagon; b_0 – lateral displacement of the center of mass of the freight from the longitudinal axis of symmetry of the wagon; l_m – base of the wagon; h – half the distance between the wheel circles; f_κ – static deflection from the tare weight of the wagon; «+» – from the side of the overloaded cart; « - » – from the side of the unloaded cart.

Since the forces P_i are parallel, the resultant is equal to their algebraic sum. After substitution of expressions for H_{0iR}, H_{0iL} and z_i, into the formulas for determining P_i, summing up the forces P_i and representing $H_{i\kappa R}$ and $H_{i\kappa L}$ in the form $\Delta H(t)$ (the function of the difference between the profile of one rail line and the profile of another rail line), the following equations are obtained:

a) with a longitudinal displacement of the center of mass of the freight:

$$\sum_{i=1}^{2n} P_i = -z_i\sum_{i=1}^{2n} S_i + S_i\left(\sum_{i=1}^{n} f_1 + \sum_{i=1}^{n} f_3\right) + Q_\eta \cos\alpha\left[\sum_{i=1}^{n} S_i(l + x_0) - \sum_{i=1}^{n} S_i(l - x_0)\right] + \\ + \frac{1}{2}\left(\sum_{i=1}^{2n} H_i S_i + \sum_{i=1}^{2n} \Delta H_i S_i\right)$$

b) with a lateral displacement of the center of mass of the freight:

$$\sum_{i=1}^{2n} P_i = -z_i\sum_{i=1}^{2n} S_i + S_i\left(\sum_{i=1}^{n} f_1 + \sum_{i=1}^{n} f_2\right) + \psi_\zeta \cos\beta\left[\sum_{i=1}^{n} (\frac{B}{2} - b_0)S_i - \sum_{i=1}^{n} (\frac{B}{2} + b_0)S_i\right] + \\ + \frac{1}{2}\left(\sum_{i=1}^{2n} H_i S_i + \sum_{i=1}^{2n} \Delta H_i S_i\right) \qquad (7)$$

c) with simultaneous longitudinal and transverse displacement of the center of mass of the freight:

$$\sum_{i=1}^{2n} P_i = -z_i \sum_{i=1}^{2n} S_i + S_i\left(f_1 + f_2 + f_3 + f_4\right) + \psi_\zeta \cos\beta \left[\sum_{i=1}^{n} (\frac{B}{2} - b_0) S_i - \sum_{i=1}^{n} (\frac{B}{2} + b_0) S_i \right] + \\ + Q_\eta \cos\alpha \left[\sum_{i=1}^{n} (l + x_0) S_i - \sum_{i=1}^{n} (l - x_0) S_i \right] + \frac{1}{2}\left(\sum_{i=1}^{2n} H_i S_i + \sum_{i=1}^{2n} \Delta H_i S_i\right) \qquad (8)$$

where H_i – longitudinal profile of the rail lines under the right or left wheels, $n=2$.

The resistance force of the vibration dampers is taken proportional to the speed of movement of the spring sets, that is, proportional to $\dot{H}_i - \dot{z}_i$.

In this case, the resistance forces will be equal to:

$$F_1 = k_i(\dot{H}_{01R} - \dot{z}_1); \quad F_2 = k_i(\dot{H}_{01L} - \dot{z}_2); \quad F_3 = k_i(\dot{H}_{01R} - \dot{z}_3); \quad F_4 = k_i(\dot{H}_{01L} - \dot{z}_4).$$

Since the forces F_i are parallel, the resultant is equal to their algebraic sum. After substituting the expressions for H_{0iR}, H_{0iL} and z_i into the formulas for F_i, summing the forces F_i and representing $\dot{H}_{i\kappa\kappa}$ or $\dot{H}_{i\kappa\kappa}$ in the form $\Delta\dot{H}(t)$), the following equations are obtained:

a) with longitudinal displacement of the center of mass of the freight:

$$\sum_{i=1}^{2n} F_i = -\dot{z}_i \sum_{i=1}^{2n} k_i + \dot{\theta}_\eta \left[\sum_{i=1}^{n} k_i (l + x_0) - \sum_{i=1}^{n} k_i (l - x_0)\right] + \frac{1}{2}\left(\sum_{i=1}^{2n} k_i \dot{H}_i + \sum_{i=1}^{n} k_i \Delta\dot{H}_i\right) \qquad (9)$$

b) with lateral displacement of the center of mass of the freight:

$$\sum_{i=1}^{2n} F_i = -\dot{z}_i \sum_{i=1}^{2n} k_i + \dot{\psi}_\zeta \cos\beta \left[\sum_{i=1}^{n} k_i (\frac{B}{2} - b_0) - \sum_{i=1}^{n} k_i (\frac{B}{2} + b_0)\right] + \frac{1}{2}\left(\sum_{i=1}^{2n} k_i \dot{H}_i + \sum_{i=1}^{2n} k_i \Delta\dot{H}_i\right) \qquad (10)$$

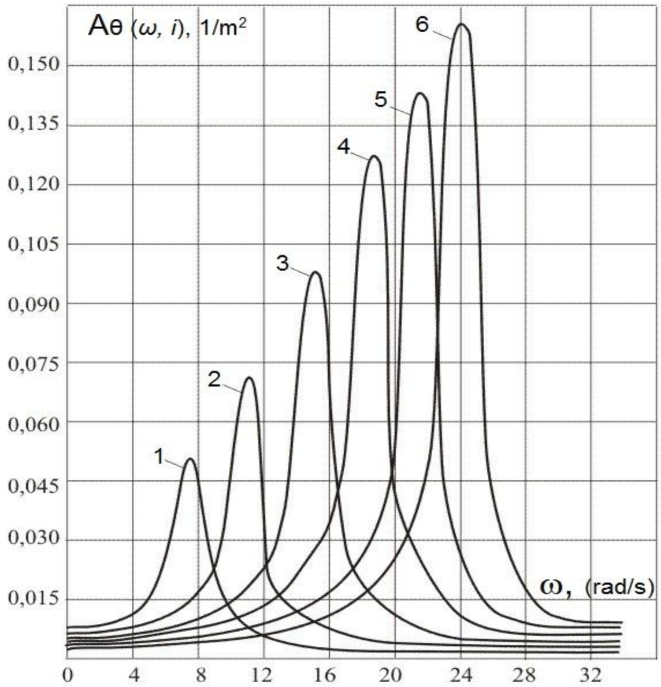
c) simultaneous longitudinal and lateral displacement of the center of mass of the freight:

$$\sum_{i=1}^{2n} F_i = -\dot{z}_i \sum_{i=1}^{2n} k_i + \dot{\psi}_\zeta \cos\beta \left[\sum_{i=1}^{n} k_i (\frac{B}{2} + b_0) - \sum_{i=1}^{n} k_i (\frac{B}{2} + b_0)\right] + \dot{\theta}_\eta \cos\alpha \times \\ \times \left[\sum_{i=1}^{n} k_i (l + x_0) - \sum_{i=1}^{n} k_i (l - x_0)\right] + \frac{1}{2}\left(\sum_{i=1}^{2n} k_i \dot{H}_i + \sum_{i=1}^{2n} k_i \Delta\dot{H}_i\right) \qquad (11)$$

Thus, the compiled design model of a wagon with an asymmetrically located freight and the equations obtained in an analytical way, oscillations of the sprung mass of a wagon with an asymmetric arrangement of the freight, characterizing the longitudinal and lateral displacement of the center of mass of the freight, can be used for mathematical modeling of forced vibrations and impact on the track of a wagon with an asymmetrically positioned freight in various operating conditions [8]. Currently, friction dampers are used in wagon bogies, in which vibration resistance is created due to dry friction between the parts of the friction damper. Friction dampers are designed so that the normal pressure between the rubbing surfaces is proportional to the amount of compression of the elastic elements of the spring suspension.

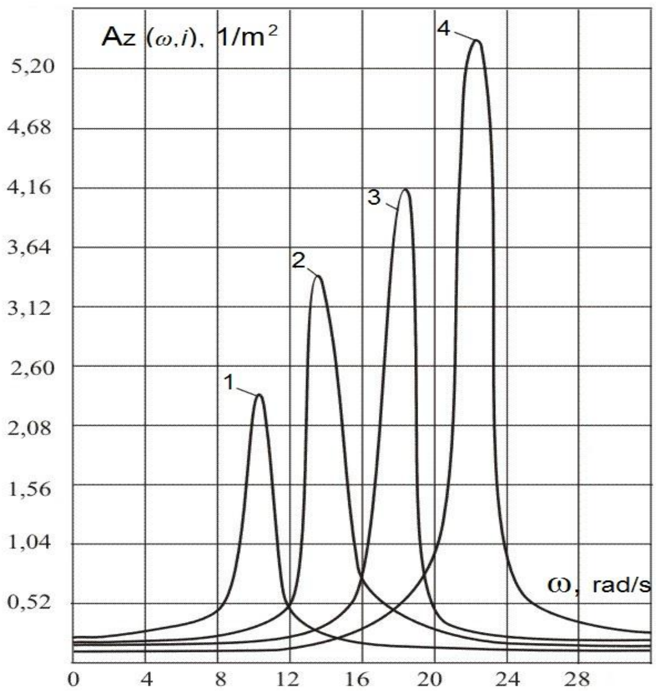
3. Results and discussion

The amplitude-frequency response was calculated as follows: first, certain values of the damper resistances were set and varied by the stiffness of the spring set, which makes it possible to establish the effect of changes in stiffness in a fairly wide range on the nature and magnitude of the amplitude-frequency response. Such dependence for pitching oscillations (see Figure 2) is given for the damper drag coefficient k, equal to 1.7 ton·sec·m^{-1}. Since the coefficient of non-simultaneity of the effect of unevenness ξ does not depend on the rigidity and degree of damping, it was not taken into account. Figure 2 shows the amplitude-frequency characteristics for bouncing oscillations, where damper resistance was taken to be equal to k = 1 ton·sec·m^{-1} and the body mass $m_к$ = 7.65 ton·sec^2·m^{-1}.



Damper resistance:
k = 1.7 ton·sec·m^{-1}; 1,2,3,4,5,6 – is the for the rigidity of the spring suspension, respectively 100; 200; 300; 400; 600; 800; 1000 ton/m;

Figure 2. Frequency response of galloping oscillations.

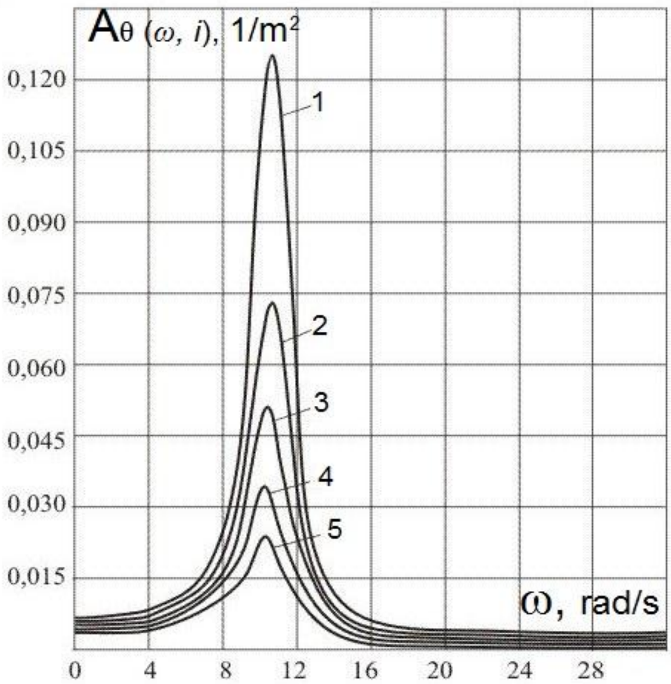


Damper resistance:
k = 1.0 ton·sec·m^{-1}; 1,2,3,4 – is the for the rigidity of the spring suspension, respectively 200; 400; 600 ton/m;

Figure 3. Frequency response of bouncing oscillations.

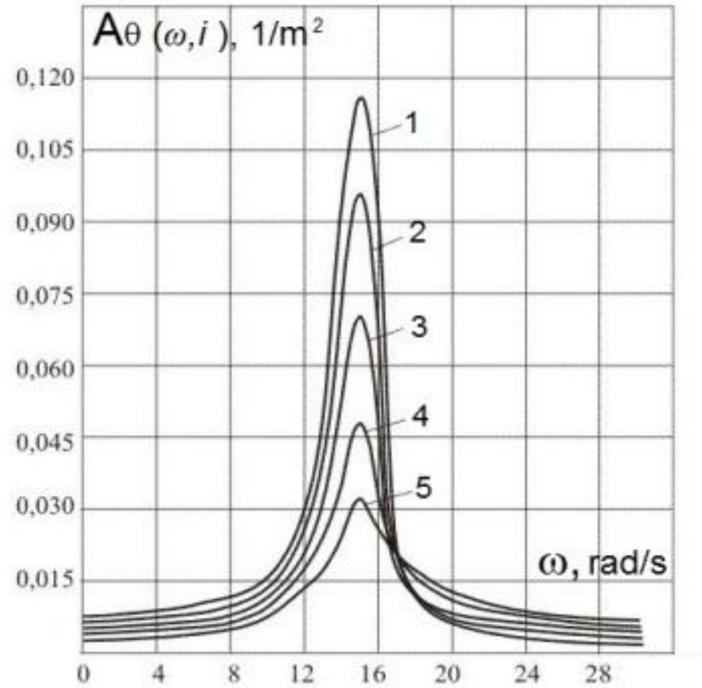
Thus, the graphical dependencies (see Figure 2 and Figure 3) show that the best conditions for the interaction of the wagon and the railway track will be provided if the vibration frequency range at which the modulus of the amplitude-frequency characteristic reaches the highest value (for each of the stiffness's), will not coincide with the frequencies of the maximum value of the spectral density of the impact of irregularities in the path. If the longitudinal profile of the track had only long irregularities with a low frequency of impact, then it would be possible to use an even more rigid spring suspension, for example, with stiffness of up to 900 ton/m. If there are mainly short irregularities along the way, it is necessary to use flexible spring suspension. If we take into account that with an increase in the speed of movement, the frequency of exposure to long irregularities increases, then, of course, the advantage remains for a more flexible spring suspension. Considering that the real path contains both short and long irregularities, it would be most expedient to be able to change the spring suspension stiffness depending on the frequency of irregularities [9].

Let us consider how the absorber's resistance affects the amplitude-frequency characteristic of the oscillations. The graphs for galloping oscillations (see Figure 4 and Figure 5) are given for spring suspension with a stiffness of 200, 400, 600, 1000 ton/m. The stiffness value equal to 400 ton/m, corresponds to the stiffness of the spring set of the TsNII-X3 bogie. The coefficient of resistance of the damper of the spring set in the calculations varied from 1 до 8 ton·sec·m^{-1}. Similar dependencies (see Figure 6) are given for bouncing oscillations at Si = 400 ton/m. From these graphs it follows that an increase in the resistance of the dampers leads to a decrease in the maximum value of the amplitude-frequency characteristic; this occurs most intensively in the case of a more rigid spring suspension. An increase in the degree of damping has an almost imperceptible effect on the amplitude-frequency response in the pre-resonant and over-resonant regions. Analysis of the change in the maximum value of the amplitude-frequency response depending on the resistance of the dampers at different stiffness of spring suspension (see Figure 7) shows that with an increase in the resistance of the dampers at the same stiffness of the spring suspension, the intensity of the decrease in the maximum values of the frequency characteristics decreases. It becomes obvious that it is possible to choose a rational drag coefficient for a given spring suspension stiffness since from a certain moment a further increase in damping practically begins to exert less and less influence.



Spring suspension stiffness 200 ton/m; 1,2,3,4,5 – is the for damper resistance, respectively 1.0; 1.7; 2.5; 3.5; 5.0 ton·sec·m^{-1};

Figure 4. Plots of the amplitude-frequency characteristic of galloping oscillations.

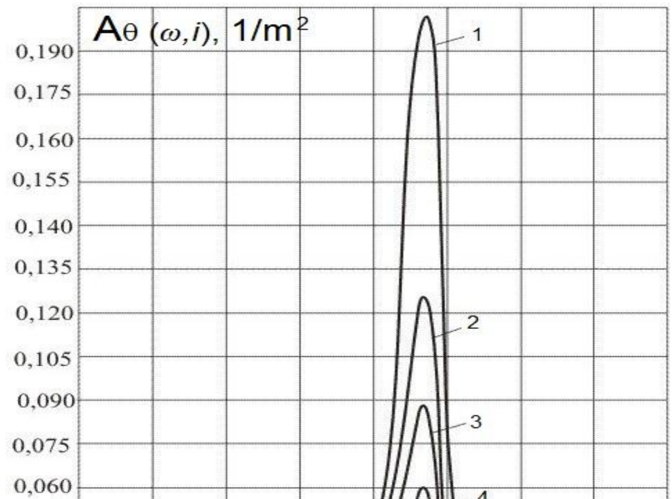


Spring suspension stiffness 400 ton/m; 1,2,3,4,5 – is the for damper resistance, respectively 1.0; 1.7; 2.5; 3.5; 5.0 ton·sec·m^{-1};

Figure 5. Plots of the amplitude-frequency characteristic of bouncing oscillations.

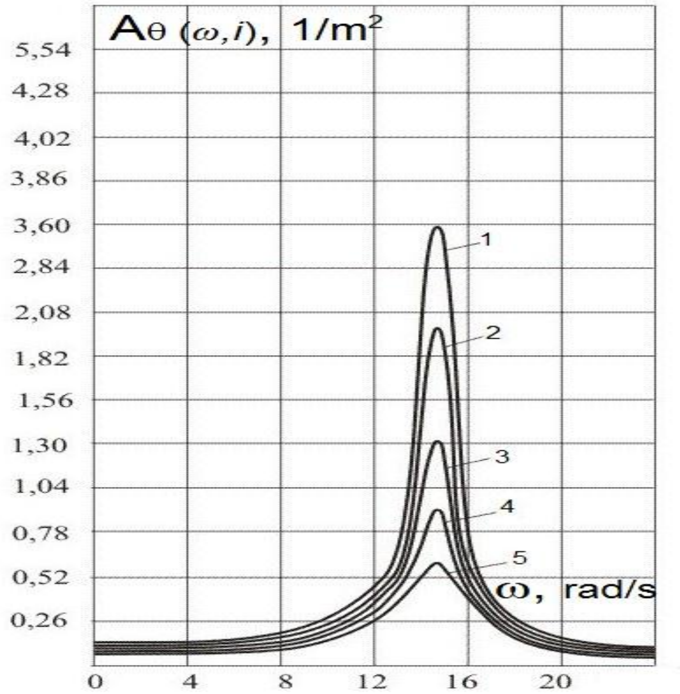
The obtained dependences allow us to make another equally important conclusion that, based on the given value of the amplitude-frequency characteristic, it is possible to determine how many times the value of the damper resistance should be less for a flexible spring suspension than for a more rigid one, and vice versa. For example, with $A_\theta(\omega)$ = 0,13 1/m^2 (see Figure 7) the value of *k* at *Si* = 1000 ton/m should be 1.6 times greater than at *Si* = 400 ton/m.

The graph, built according to the results of calculations (see Figure 8), shows the change in the maximum values of the amplitude-frequency characteristic depending on the stiffness of the spring suspension at various resistances of the damper.



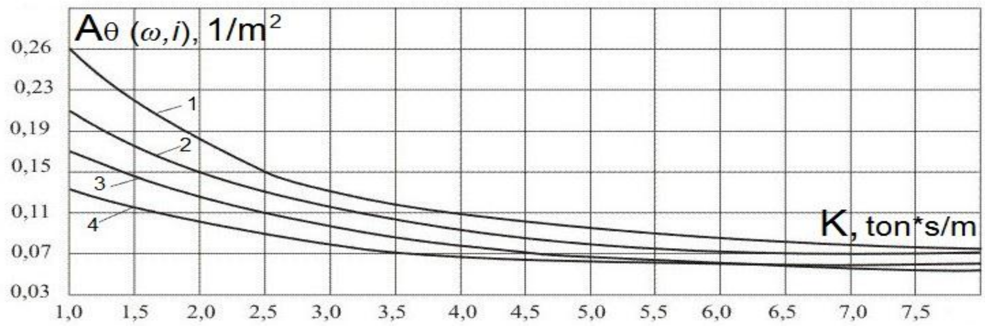
Spring suspension stiffness 600 ton/m; 1,2,3,4,5 – is the for damper resistance, respectively 1.0; 1.7; 2.5; 3.5; 5.0 ton·sec·m^{-1};

Figure 6. Plots of the amplitude-frequency characteristic of galloping oscillations.



Spring suspension stiffness 400 ton/m; 1,2,3,4,5 – is the for damper resistance, respectively 1.0; 1.7; 2.5; 3.5; 5.0 ton·sec·m^{-1};

Figure 7. Plots of the amplitude-frequency characteristic of bouncing oscillations.



1,2,3,4 – is the with the stiffness of the spring suspension, respectively 1000; 600; 400; 200 ton/m;

Figure 8. The graph of the dependence of the maximum value of the amplitude frequency characteristic of the pitching oscillations on the value of the damper resistance.

4. Conclusion

It can be seen from the figure that an increase in resistance of more than 6 ton·sec·m^{-1} becomes impractical. It should be kept in mind that the excess resistance of vibration dampers introduced into the spring suspension adversely affects the frequency response in the resonant zone. Thus, the resistance of the vibration damper must be selected proceeding from the condition of a significant decrease in the values of the amplitude-frequency characteristic in the resonance zone and their insignificant increase in the resonance zone. It was found that an increase in the resistance of the dampers leads to a decrease in the maximum value of the amplitude-frequency characteristic; this occurs most intensively in the case of a more rigid spring suspension. An increase in the degree of damping has an almost imperceptible effect on the amplitude-frequency response in the pre-resonant and over-resonant regions. The performed analysis of the change in the maximum value of the amplitude-frequency characteristic depending on the resistance of the dampers at different stiffness of spring suspension shows that with an increase in the resistance of the dampers at the same stiffness of the spring suspension, the intensity of the decrease in the maximum values of the frequency characteristics decreases. The dependences obtained as a result of the studies performed allow us to make another equally important conclusion that, based on the given value of the amplitude-frequency characteristic, it is possible to determine how many times the value of the damper resistance should be less for a flexible spring suspension than for a more rigid one, and vice versa.

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