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Researching the Vibration Characteristics of Suspension System of Automotive In the Frequency Range

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Abstract. The design parameters of the automotives not only affect the ride comfort but also affect the durability of vehicle structures, especially in the low frequency region. To enhance the automotive performance, a quarter dynamic model of automotive is used to establish the vibration equations in the time region. The vibration equations in the time region are then transformed to the frequency region based on the Laplace transformation to calculate the amplitude-frequency and acceleration-frequency characteristics of automotives. The effect of the design parameters of the automotive and suspension system such as the damping coefficient, stiffness coefficient, and mass of the automotive are then simulated and analyzed, respectively. The research result shows that both the amplitude-frequency and acceleration-frequency responses of the automotive are remarkably affected by the design parameters of the automotives in the frequency region. To improve the ride comfort and enhance the structural durability of the automotive, during the automotive design, the damping coefficient needs to enhance while both the stiffness coefficient and automotive mass needs to be optimized to reduce both the resonance amplitudes and resonant frequencies of the automotive.

Keywords: Low frequency region, ride comfort, automotive dynamic model, optimal design.

1. Introduction

To enhance the ride comfort and health of the driver, the optimization of the automotive dynamic parameters as well as control the damping coefficient of the automotive suspension system had been researched and developed [1-7]. Two indexes of the root-mean-square (RMS) and power-spectrum-density (PSD) acceleration responses of the vehicle were used to evaluate the results. The research results showed that the driver's ride comfort had been significantly improved under various operating conditions. The effect of the design parameters of the automotive suspension such as the stiffness and damping coefficients of the suspension systems of the vehicle, wheel, cab, and driver's seat were analyzed under the different moving speeds and various road surfaces of the vehicles [3, 6, 8-9]. The results also showed that the dynamic parameters of the vehicle isolation system significantly affected on the driver's ride comfort. However, in all of the above studies, the effect of the design parameters of the vehicles mainly evaluated based on the automotive dynamic models and the mathematical equations of the automotives in the time region to analyze the ride comfort of the automotive as well as the cab and driver's seat. Based on the ISO 2631-1 [10], the vehicle's ride comfort was not only evaluated in the region but also concerned in the frequency region. Therefore, the ride comfort in the frequency region has not yet concerned in the existed researches.

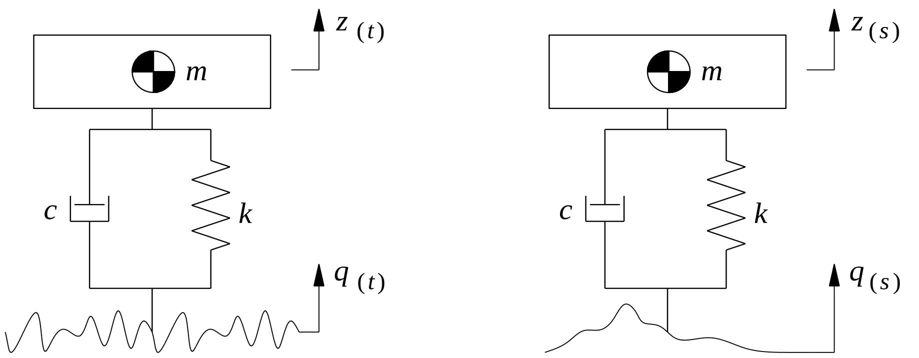
The performance of vehicles was not only evaluated via the ride comfort index but also evaluated via the durability of vehicle structures [11-13]. To evaluate the durability of the suspension system of vehicles, the interaction model of the wheel suspension system and road surface was used [14-16]. Besides, the dynamic model of the fully vehicle was also used [3, 5]. However, the vibration response of the vehicles was mainly computed in the time region. Then, the FFT method was mainly used to calculate the PSD acceleration-frequency response of the vehicles due to their mathematical equations being extremely complicated. Although the amplitude-frequency and acceleration-frequency characteristics of the automotive in the frequency region could be evaluated, however, the FFT method is difficult to understand well the correlation between the vehicle design parameters and the amplitude-frequency characteristics in the frequency region to design and optimize the ride comfort as well as the durability of vehicle structures.

To clarify this issue, a quarter dynamic model of automotive is used to establish the vibration equations in the time region. Based on the Laplace transformation [17], the vibration equations in the time region are then transformed to the frequency region. The effect of the design parameters of the automotive and suspension system such as the damping coefficient, stiffness coefficient, and mass of the automotive on the amplitude-frequency and acceleration-frequency responses of the automotive in the frequency region are then simulated and analyzed, respectively.

The major goal of this study is to evaluate the influence of the design parameters of automotives on the ride comfort and reliability of the automotive in the time and frequency regions.

2. The vibration dynamics model of the simple automotive

The vibration characteristic in the frequency region of the automotives had been analyzed based on the 2D and 3D automotive dynamic models [3, 5]. However, the FFT method was mainly used to calculate the PSD acceleration-frequency response of the vehicles due to their mathematical equations being extremely complicated. This study mainly evaluates the influence of dynamic parameters of the automotive such as the vehicle mass, stiffness and damping parameters in the frequency domain and elucidates the calculation method of the automotive vibration in the frequency. Thus, a quarter automotive dynamic model using give in Figure 1 is applied to calculate the results.



(a) The model in the time region (b) The model in the frequency region

Figure 1. The simple dynamic model of the automotive.

where *m* is the mass of the automotive, *c* is the damping coefficient, *k* is the stiffness coefficient of the suspension system, $z(t)$ is the displacement of the automotive, and *q* is the vibration excitation of the suspension system.

Based on the automotive vibration model in Figure1 and Newton second law, it vibration equation in the time region has been expressed as follows:

$$m\ddot{z}(t) = c[\dot{q}(t) - \dot{z}(t)] + k[q(t) - z(t)] \tag{1}$$

Equation (1) is then rewritten by:

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = c\dot{q}(t) + kq(t) \tag{2}$$

In order to calculate the vibration response of the automotive in the frequency region from the time region, the vibration variables of the displacement, speed, and acceleration of the automotive and the vibration excitation of the suspension system in the time region has been transformed to the frequency region based on the Laplace transformation as follows [17]:

$$\begin{cases} z(t) = Z(s) \\ \dot{z}(t) = sZ(s) - Z(0) \\ \ddot{z}(t) = s^2 Z(s) - sZ(0) - Z(0) \\ q(t) = Q(s) \\ \dot{q}(t) = sQ(s) - Q(0) \end{cases} \tag{3}$$

where $Z(s)$ and $Z(0)$ are the variables of the automotive displacement in the frequency region; $Q(s)$ and $Q(0)$ are the variables of the suspension displacement in the frequency region at the time t and the initial time t_0; and $s = i\omega$ ($\omega = 2\pi f$, $i^2 = -1$, and $s^2 = (i\omega)^2 = -\omega^2$) is a complex variable in the frequency domain.

By replacing Equation (3) into Equation (2), we obtain:

$$m\{s^2 Z(s) - sZ(0) - Z(0)\} + c\{sZ(s) - Z(0)\} + kZ(s) = c\{sQ(s) - Q(0)\} + kQ(s) \tag{4}$$

Assuming that at the initial time of the excitation $t = 0$, the value of $Z(0)$ is defined by $Z(0) = 0$, thus, Equation (4) is re-calculated and written as follows:

$$ms^2 Z(s) + csZ(s) + kZ(s) = csQ(s) + kQ(s) \tag{5}$$

By transforming Equation (5) with a ratio of $H(s) = Z(s)/Q(s)$, we obtain:

$$H(s) = \frac{Z(s)}{Q(s)} = \frac{cs + k}{ms^2 + cs + k} \tag{6}$$

Therefore, Equation (6) performs the transfer function of the vibration excitation $Q(s)$ and the displacement $Z(s)$ in the frequency region, and $H(s)$ is defined as the vibration transfer function in the frequency region.

With the excitation frequency of $\omega = 2\pi f$, $s = i\omega$, $i^2 = -1$, and $s^2 = (i\omega)^2 = -\omega^2$, Equation (6) is written as follows:

$$H(i\omega) = \frac{Z(i\omega)}{Q(i\omega)} = \frac{k + i\omega c}{(k - m\omega^2) + i\omega c} \tag{7}$$

By mathematically transforming Equation (7), the vibration transfer function $H(i\omega)$ has been determined by:

$$H(i\omega) = \frac{1 + i\omega \frac{c}{k}}{\left(1 - \omega^2 \frac{m}{k}\right) + i\omega \frac{c}{k}} = \frac{1 + 2i\omega \frac{c}{2\sqrt{km}\sqrt{\frac{k}{m}}}}{\left(1 - \omega^2 / \left(\frac{k}{m}\right)\right) + 2i\omega \frac{c}{2\sqrt{km}\sqrt{\frac{k}{m}}}} \tag{8}$$

Let $\omega_0 = \sqrt{\frac{k}{m}}$ and $\zeta = \frac{c}{2\sqrt{km}}$, we have:

$$H(i\omega) = \frac{1 + 2i\zeta \dfrac{\omega}{\omega_0}}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2 + 2i\zeta \dfrac{\omega}{\omega_0}} \quad (9)$$

Let $\lambda = \omega / \omega_0$, Equation (9) is rewritten by:

$$H(i\omega) = \frac{1 + 2i\zeta\lambda}{1 - \lambda^2 + 2i\zeta\lambda} \quad (10)$$

Let $a = 1 - \lambda^2$ and $b = 2\zeta\lambda$, Equation (10) is rewritten by:

$$H(i\omega) = \frac{1 + ib}{a + ib} = \frac{(1+ib)(a-ib)}{(a+ib)(a-ib)} = \frac{a - ib + iab - i^2b^2}{a^2 - i^2b^2} = \frac{a + b^2}{a^2 + b^2} + i\frac{b(a-1)}{a^2+b^2}, \; (i^2 = -1) \quad (11)$$

Therefore, the amplitude of the transfer function $H(i\omega)$ is determined by:

$$|H(i\omega)| = \sqrt{\left(\frac{a+b^2}{a^2+b^2}\right)^2 + \left(\frac{b(a-1)}{a^2+b^2}\right)^2} = \sqrt{\frac{a^2 + 2a^2b^4 + b^4 + a^2b^2 - 2ab^2 + b^2}{(a^2+b^2)^2}} = \sqrt{\frac{1+b^2}{a^2+b^2}} \quad (12)$$

By replacing $a = 1 - \lambda^2$ and $b = 2\zeta\lambda$ into Equation (12), we have:

$$|H(i\omega)| = \sqrt{\frac{1 + (2\zeta\lambda)^2}{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \quad (13)$$

The acceleration-frequency response of the model in the frequency region is determined by:

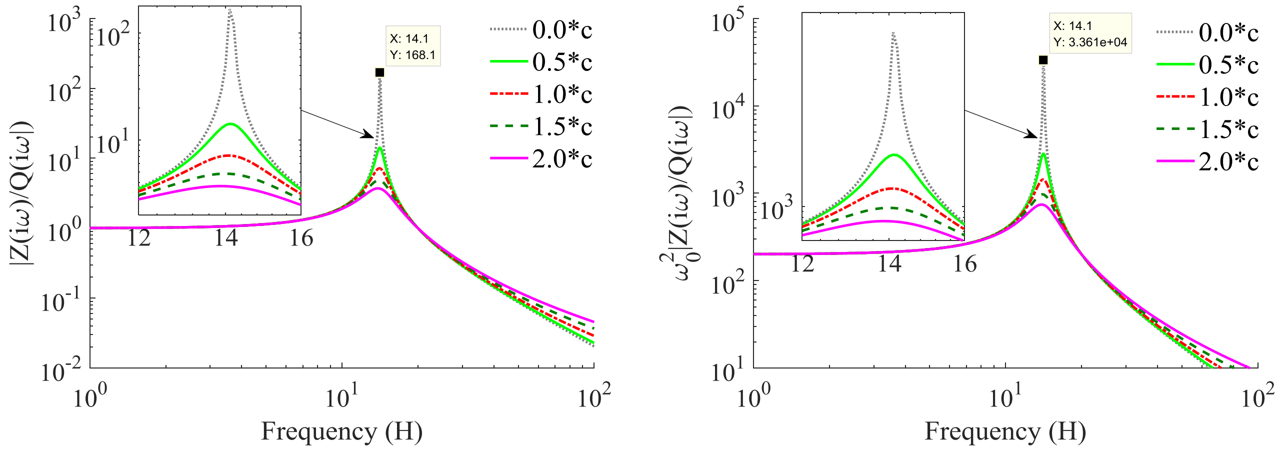
$$|\ddot{H}(i\omega)| = \omega^2 |H(i\omega)| = \omega^2 \sqrt{\frac{1 + (2\zeta\lambda)^2}{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \quad (14)$$

Equations (13-14) are then applied to simulate and evaluate the effect of the design parameters of the automotive and suspension on the amplitude-frequency and acceleration-frequency responses of the automotive in the frequency region.

3. The simulation and analysis result

3.1. The effect of the damping coefficient

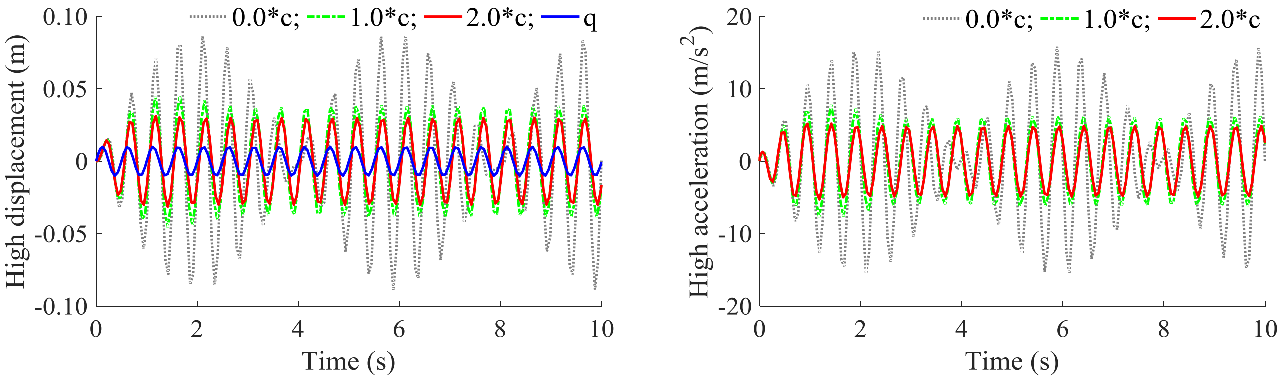
To evaluate the effect of the damping coefficient on the amplitude-frequency and acceleration-frequency responses of the automotive in the frequency region, based on the design parameters of automotives including m = 500 kg, $c = 10^3$ Ns/m, and $k = 10^5$ N/m, the different values of C = [0.0×c, 0.5×c, 1.0×c, 1.5×c, 2.0×c] are simulated. The simulation results are plotted in Figure 2.



(a) Amplitude - frequency (b) Acceleration - frequency

Figure 2. The effect of the damping coefficient on the vibration response of the automotive suspension system in the frequency region.

The results of the amplitude-frequency and acceleration-frequency responses of the automotive in Figure 2a and Figure 2b show that the resonant frequency of the dynamic model of the automotive appears at f = 14.1Hz. This is due to the influence of the resonant frequency of the suspension model $f = f_0 = \sqrt{k/m} = \sqrt{10^5/500} = 14.14$ Hz. Thus, this result implies that during the design process of the automotive suspension system, the mass m of the automotive body and stiffness coefficient k of the suspension system need be concerned to avoid resonance at a certain frequency. This issue will be analyzed in Section 3.2 and 3.3. In the case of the suspension system without the damping coefficient (C = 0.0×c), both the amplitude-frequency and acceleration-frequency responses are strongly increased. This means that the ride comfort of the vehicle is greatly reduced. With the damping coefficient C is increased by 0.5×c, 1.0×c, 1.5×c, and 2.0×c, both the amplitude-frequency and acceleration-frequency responses are also reduced, thus, the ride comfort of the vehicle is improved, especially with C = 2.0×c. Additionally, the effect of the damping coefficient on the automotive vibration response in the time region is also given in Figure 3.



(a) Amplitude - frequency (b) Acceleration - frequency

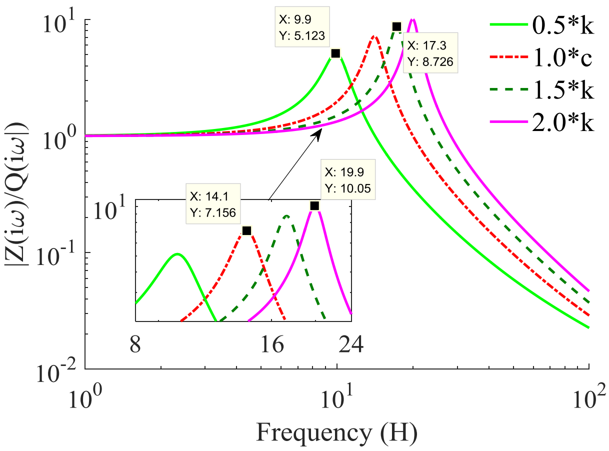
Figure 3. The effect of the damping coefficient on the vibration response of the automotive suspension system in the time region.

Observing both Figure 3a and Figure 3b, we can see that the high vibration of both the displacement and acceleration of the automotive are remarkably reduced with the increase of the damping coefficient, especially at 2.0×c. This is reason that the automotive suspension system should be equipped with the damper with the passive, semi-active or active damping coefficient [1-2, 6-7] to further improve the ride comfort of the automotives.

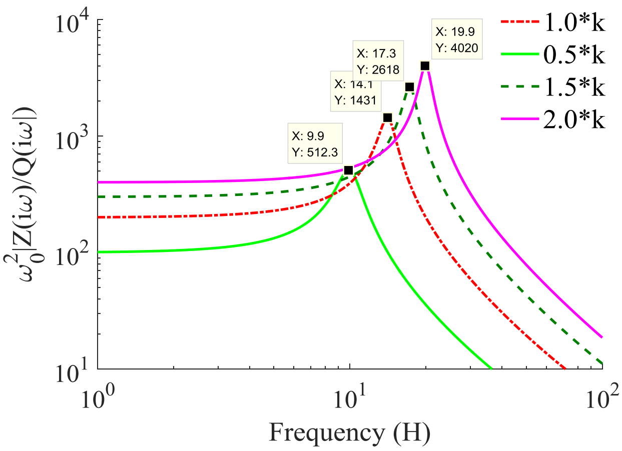
3.2. The effect of the stiffness coefficient

To evaluate the effect of the stiffness coefficient on the amplitude-frequency and acceleration-frequency responses of the automotive in the frequency region, a change range of the stiffness coefficient from K = [0.5×k, 1.0×k, 1.5×k, 2.0×k] are computed and plotted in Figure 4, respectively.

The simulation results show that under the effect of the stiffness coefficients of the automotive suspension system including K = 0.5×k, K = 1.0×k, K = 1.5×k, and K = 2.0×k, the resonant frequencies of the dynamic model of the automotive appear at 9.9Hz, 14.1Hz, 17.3Hz, and 19.9Hz on both the amplitude-frequency and acceleration-frequency responses of the automotive. This also is due to the influence of the resonant frequency of the suspension model with the change of the stiffness coefficients, respectively, such as $f_{01} = \sqrt{0.5\times k/m} = \sqrt{0.5\times 10^5/500} = 10$ Hz, $f_{02} = \sqrt{1.0\times k/m} = \sqrt{1.0\times 10^5/500} = 14.14$ Hz, $f_{03} = \sqrt{1.5\times k/m} = \sqrt{1.5\times 10^5/500} = 17.32$ Hz, and $f_{04} = \sqrt{2.0\times k/m} = \sqrt{2.0\times 10^5/500} = 20$ Hz. Additionally, based on the simulation results in both Figure 4a and Figure 4b, we can see that the resonance amplitudes are also increased with the increase of the resonant frequencies. Therefore, based on the actual automotive structure, the stiffness coefficient of the automotive suspension system should be calculated and optimized to reduce the resonance amplitudes of the automotive and enhance the structural durability of the suspension systems under the different operating conditions.



(a) Amplitude - frequency

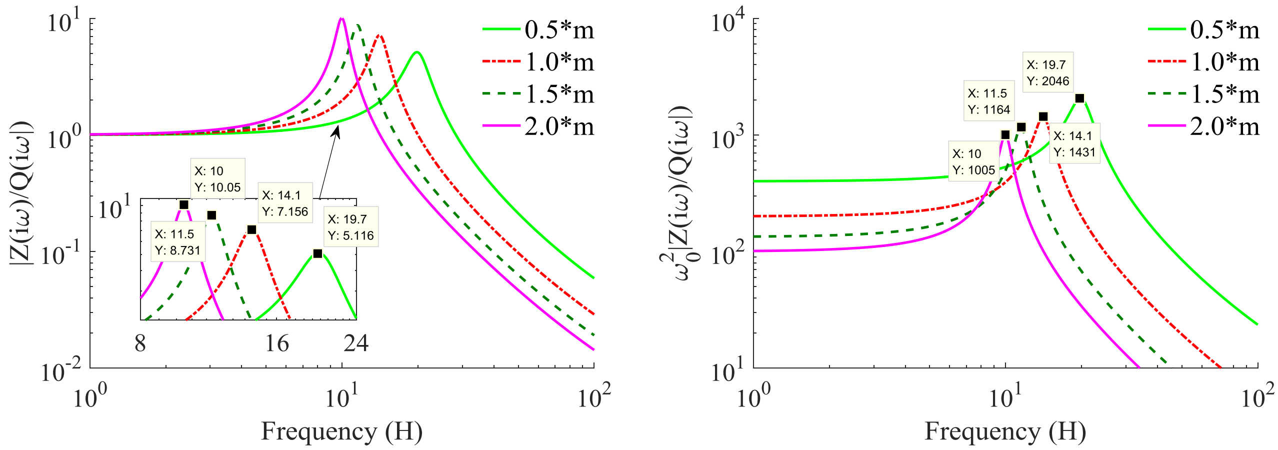


(b) Acceleration - frequency

Figure 4. The effect of the stiffness coefficient on the vibration response of the automotive suspension system in the frequency region.

3.3. The effect of the automotive mass

To evaluate the effect of the mass of the automotive body on the amplitude-frequency and acceleration-frequency responses of the automotive in the frequency region, a change range of the mass from M = [0.5×m, 1.0×m, 1.5×m, 2.0×m] are also simulated, respectively. The simulation results of the amplitude-frequency and acceleration-frequency responses in the frequency region have been shown in Figure 5a and Figure 5b, respectively.



(a) Amplitude - frequency (b) Acceleration - frequency

Figure 5. The effect of the automotive mass on the vibration response of the automotive suspension system in the frequency region.

Similarly, the simulation results show that under the effect of the mass of the automotive body including $M = 0.5\times m$, $M = 1.0\times m$, $M = 1.5\times m$, and $M = 2.0\times m$, the resonant frequencies of the automotive dynamic model appear at 19.7Hz, 14.1Hz, 11.5Hz, and 10Hz on both the amplitude-frequency and acceleration-frequency responses of the automotive. This also is due to the influence of the change of the mass m, respectively, such as $f_{01} = \sqrt{k/0.5\times m} = \sqrt{10^5/0.5\times 500} = 20$ Hz, $f_{02} = \sqrt{k/1.0\times m} = \sqrt{10^5/1.0\times 500} = 14.14$ Hz, $f_{03} = \sqrt{k/1.5\times m} = \sqrt{10^5/1.5\times 500} = 11.55$ Hz, and $f_{04} = \sqrt{k/2.0\times m} = \sqrt{10^5/2.0\times 500} = 10$ Hz. Additionally, based on the simulation results in both Figure 5a and Figure 5b, it is shown that the resonance amplitudes of the amplitude-frequency responses is increased while the resonance amplitudes of the acceleration-frequency responses is reduced with the increase of the automotive mass. Therefore, the result shows that the mass of the automotive clearly affect the amplitude-frequency characteristics of the automotive in the frequency region. To optimize both the amplitude-frequency and acceleration-frequency responses under the change of the automotive mass, the mass of the automotive in the design process should also be calculated and optimized to reduce the resonance amplitudes of the automotive as well as enhance the structural durability under the various working conditions.

4. Conclusion

A simple model and calculation method of the automotive dynamic that can reflect the influence of all the suspension design parameters of automotive based on the Laplace transformation is proposed and computed in this study.

Both the amplitude-frequency and acceleration-frequency responses of the automotive are remarkably affected by the damping coefficient in the frequency region. To improve the ride comfort of the automotives, the damping coefficient must be added on the automotive suspension systems.

Bothe the mass m of the automotive body and stiffness coefficient k of the automotive suspension system greatly affect both the resonance amplitudes and resonant frequencies of the automotive. To improve the ride comfort and enhance the structural durability of the automotive, during the automotive design, a change range of both the mass m and stiffness coefficient k needs to be analyzed and optimized to reduce both the resonance amplitudes and resonant frequencies of the automotive.

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