

Study the service strokes in professional tennis

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Abstract. This article examines one of the most important strokes in the tennis sport, namely, service strokes. The study covers three types of strokes: flat, slice and kick service strokes. A specially made program has been compiled in the area of MatLab, with the help of which the strongly nonlinear differential equations are integrated. These equations describe the trajectory of the tennis ball under appropriate initial conditions. All types of air resistance are included. Here, the main focus is the formation of the Magnus force. Thanks to it, the tennis ball changes its trajectory depending on the degree and type of rotation. The article is intended for a wide range of readers: scientists, tennis players, and tennis coaches.

Keywords: tennis, service strokes, 3D motion, numerical solution, Magnus, MatLab.

1. Introduction

Tennis service stroke is one of the most powerful weapons in modern competitive tennis. Every tennis match begins with this stroke. With the knowledge in this study about the service strokes, the tennis player can turn this shot into a true advantage for his game. Properly executed, this stroke is very difficult to return and makes it very difficult for the opponent. In recent years, there has been a significant increase in studies related to the motion and flight of a variety of sports balls in a real fluid environment, including tennis balls. For example, such studies could be seen in the works [1-7]. The effect of rotation of the tennis ball, the so-called spin, as well as the emerging force of Magnus, using experimental and computational methods, can be read in the publication [8]. Service strokes do not differ from basic strokes, such as forehand and backhand, in terms of kinematics and dynamics, [9-10]. However, these strokes are characterized by some peculiarities, [11-13]. First, their linear velocity is significantly higher. As for their angular velocity, it is relatively lower. The three main types of services in competitive professional tennis have been studied: flat, slice and kick. The main goal of this work is to study numerically the trajectories of the three types of services, flat, slice and kick, by using a suitable program of MatLab.

2. Differential equations

The tennis ball is a round, hollow, several-layered sphere with a certain average thickness. The ball is an elastic body with strong damping characteristics. The air inside the competition balls is under pressure higher than atmospheric. A fluffy woolen knit is glued on the rubber surface of the ball (Figure 1 and Figure 2). The ball is assumed to be a perfectly rigid hollow closed two-layer spherical shell. The rubber wall is defined by an inner radius $r = 0.026 \text{ m}$ and an outer radius $R_1 = 0.030 \text{ m}$. Wool knit gives the total size of the ball with a radius $R = 0.0325 \text{ m}$. The main physical characteristics of the tennis ball are: the mass of the tire $m_1 = 0.045 \text{ kg}$, the mass of the knit $m_2 = 0.012 \text{ kg}$, the total mass $m = m_1 + m_2$, and the mass moment of inertia $J = 3.155 \times 10^{-5} \text{ kg} \cdot \text{m}^2$.

The flight of a tennis ball is represented as a general motion of a hollow spherical rigid body in an air environment, the influence of which is taken into account with aerodynamic forces.

The law of motion of the tennis ball represents the six functions: $x(t)$, $y(t)$, $z(t)$, $\psi(t)$, $\theta(t)$ and $\varphi(t)$, (see Figure 1). They are included in the both vectors:

$$\mathbf{r} = \langle x(t) \ y(t) \ z(t) \rangle^T \quad (1)$$

$$\mathbf{p} = \langle \psi(t) \ \theta(t) \ \varphi(t) \rangle^T \quad (2)$$

The rotation speed of a tennis ball is described by the angular velocity vector:

$$\boldsymbol{\omega} = \langle \omega_x \ \omega_y \ \omega_z \rangle^T \quad (3)$$

The two vectors \mathbf{p} and $\boldsymbol{\omega}$ are related with the following kinematical equation:

$$\boldsymbol{\omega} = \mathbf{S} \cdot \mathbf{p} \quad (4)$$

The matrix \mathbf{S} , when the Cardan angles are used, has the form:

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & \sin \theta \\ 0 & \cos \psi & -\cos \theta \cdot \sin \psi \\ 0 & \sin \psi & \cos \theta \cdot \cos \psi \end{bmatrix} \quad (5)$$

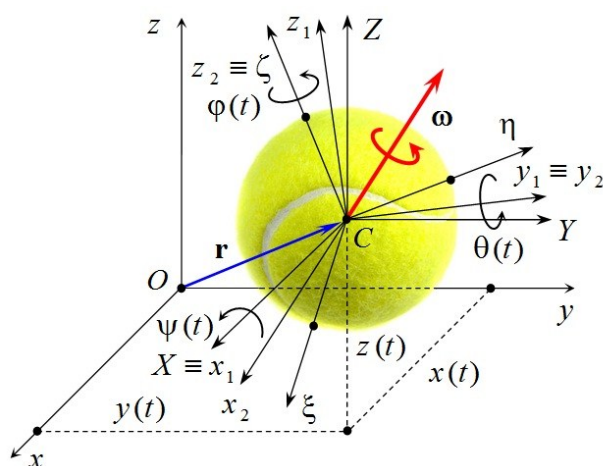


Figure 1. Generalized coordinates.

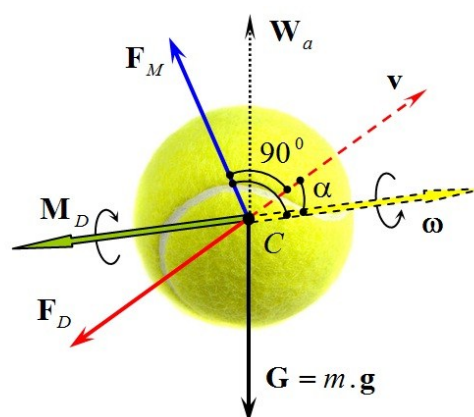


Figure 2. Aerodynamic forces.

The system of differential equations, presented in vector-matrix form, which determines the law of motion of the tennis ball, has the form:

$$m \cdot \ddot{\mathbf{r}} = \mathbf{F} \quad (6)$$

$$J \cdot (\mathbf{S} \cdot \ddot{\mathbf{p}} + \dot{\mathbf{S}} \cdot \dot{\mathbf{p}}) = \mathbf{M} \quad (7)$$

where \mathbf{F} and \mathbf{M} are the main force and the main moment of all forces acting on the tennis ball. They are reduced to its mass center C , (Figure 2). The main force \mathbf{F} is a vector sum of three forces:

$$\mathbf{F} = \mathbf{F}_g + \mathbf{F}_D + \mathbf{F}_L \quad (8)$$

The force \mathbf{F}_g is a vector sum between the weight of the tennis ball \mathbf{G} and the lifting force \mathbf{W}_a according to Archimedes law, (Figure 2). It is determined by the formula:

$$\mathbf{F}_g = \mathbf{G} + \mathbf{W}_a = (m - V \cdot \rho) \cdot \mathbf{g} \quad (9)$$

where V is the volume of the tennis ball, ρ is the density of the air, and \mathbf{g} is the ground acceleration.

The density of the air is strongly influenced by temperature. For this study, the air density is taken as the average value $\rho = 1.205 \text{ kg/m}^3$ at $t = 20^\circ \text{C}$. The earth acceleration vector has the form:

$$\mathbf{g} = \langle 0 \quad 0 \quad -g \rangle^T \quad (10)$$

and for average latitudes, the magnitude of earth acceleration is assumed $g = 9.81 \text{ m/s}^2$.

In fact, for the tennis balls, the Archimedes lift force can be neglected. The force \mathbf{F}_D is the resistance force, (Figure 2). It is one of the main aerodynamic forces. It is determined by the formula:

$$\mathbf{F}_D = \frac{1}{2} \cdot C_D \cdot A \cdot \rho \cdot |\dot{\mathbf{r}}| \cdot \dot{\mathbf{r}} \quad (11)$$

where A is the area of the middle section of the tennis ball.

The drag coefficient C_D is considered to be a turbulent flow regime. When the tennis balls are flying at medium velocities, this coefficient retains a relatively constant value for a wide range of Reynolds numbers, namely $5 \times 10^4 \leq R_e \leq 7.5 \times 10^4 \text{ m}^2/\text{s}$, [13-16]. The other main aerodynamic force is the Magnus force, (Figure 2). It is determined by the formula:

$$\mathbf{F}_L = \frac{1}{2} \cdot C_L \cdot A \cdot \rho \cdot v^2 \cdot \frac{\boldsymbol{\Omega} \cdot \dot{\mathbf{r}}}{|\boldsymbol{\Omega} \cdot \dot{\mathbf{r}}|} \quad (12)$$

In formula (12), the matrix $\boldsymbol{\Omega}$ is constructed from the components of the angular velocity vector $\boldsymbol{\omega}$ and has the form:

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad (13)$$

The lift coefficient C_L of Magnus, for the same turbulent flow regime, $5 \times 10^4 \leq R_e \leq 7.5 \times 10^4 \text{ m}^2/\text{s}$, depends not only on the Reynolds numbers, but also on the spin parameter S . When the spin parameter S is increased, the value of lift coefficient C_L is also increased. This phenomena is clearly described in the works [13-16]. For this study, the spin parameter is assumed with the value:

$$S = \frac{\omega \cdot R}{v} = \frac{150 \times 0.0325}{30} = 0.1625 \quad (14)$$

The main moment coincides with the drag aerodynamic moment as follows:

$$\mathbf{M} \equiv \mathbf{M}_D = \frac{1}{2} \cdot C_M \cdot A \cdot \rho \cdot v^2 \cdot \frac{\boldsymbol{\Omega}}{|\boldsymbol{\Omega}|} \quad (15)$$

The coefficient C_M depends on the Reynolds number, the spin rate, and the condition of the uniform felt fabric of the tennis ball, and in particular, whether the balls are new or old with a thin knit.

The study, which is shown on the next pages, is performed with the following values: $C_D = 0.50$, $C_L = 0.20$, and $C_M = 0.01$.

3. Tennis service strokes

Tennis service strokes of competitors are the most diverse, but nevertheless, they can be grouped into three main types. The classification is made mainly depending on the point of contact of the tennis ball with the racket and the magnitude and direction of the rotation of the ball.

In the first type of service, called flat, the upper (topspin) and lateral (side spin) rotations are the smallest, and the average angular velocity is about $\omega = 120 \text{ s}^{-1}$, or $n = 1146 \text{ rpm}$. Due to less rotation, the tennis player achieves the greatest linear initial velocity, around $v = 70 \text{ m/s}$, or $v = 252 \text{ km/h}$.

The second type of service is called slice. It is characterized by greater rotation and the angular velocity reaches $\omega = 230 \text{ s}^{-1}$, or $n = 2196 \text{ rpm}$. Compared to the flat serve, the slice serve has one and a half times the topspin and almost twice the side spin. The trajectory of the ball is shorter and more laterally curved.

The third type of service is called kick. It is defined by the greatest rotation and the angular velocity reaches $\omega = 340 \text{ s}^{-1}$, or $n = 3245 \text{ rpm}$. It has the most topspin and the most side spin. The trajectory of the ball is the shortest and most laterally curved. The kick serve is the most difficult to execute, but it is the main weapon in the arsenal of modern tennis players.

4. Numerical solution

In the real game, each of the three types of service is executed at a different initial velocity. However, in this computer simulation, the same initial velocity is assumed for all three service strokes. This is done in order to study how rotation and the Magnus effect change the tennis ball trajectory. The following initial conditions are assumed for the three types of service strokes.

Initial position:

$$\mathbf{r}_0 = \langle x_0 = 0 \quad y_0 = 0 \quad z_0 = 2.90 \text{ m} \rangle^T \quad (16)$$

Initial linear velocity:

$$\mathbf{v}_0 \equiv \dot{\mathbf{r}}_0 = \begin{bmatrix} v_{x,0} \equiv \dot{x}_0 = 52.384 \text{ m/s} \\ v_{y,0} \equiv \dot{y}_0 = 0 \\ v_{z,0} \equiv \dot{z}_0 = -6.432 \text{ m/s} \end{bmatrix} \quad (17)$$

$$v_0 = 52.777 \text{ m/s} \equiv 190 \text{ km/h} \quad (18)$$

The following initial angular velocities are assumed for the three types of service strokes, [13]:

Flat service stroke

$$\boldsymbol{\omega}_0 = \begin{bmatrix} \omega_{x,0} \equiv \dot{\psi}_0 = 22.00 \text{ s}^{-1} \\ \omega_{y,0} \equiv \dot{\theta}_0 = 18.40 \text{ s}^{-1} \\ \omega_{z,0} \equiv \dot{\phi}_0 = 117.00 \text{ s}^{-1} \end{bmatrix} \quad (19)$$

$$\omega_0 = 120 \text{ s}^{-1} \equiv 1150 \text{ rpm} \quad (20)$$

Slice service stroke

$$\boldsymbol{\omega}_0 = \begin{bmatrix} \omega_{x,0} \equiv \dot{\psi}_0 = 29.60 \text{ s}^{-1} \\ \omega_{y,0} \equiv \dot{\theta}_0 = 73.80 \text{ s}^{-1} \\ \omega_{z,0} \equiv \dot{\phi}_0 = 214.20 \text{ s}^{-1} \end{bmatrix} \quad (21)$$

$$\omega_0 = 228 \text{ s}^{-1} \equiv 2182 \text{ rpm} \quad (22)$$

Kick service stroke

$$\boldsymbol{\omega}_0 = \begin{bmatrix} \omega_{x,0} \equiv \dot{\psi}_0 = 31.10 \text{ s}^{-1} \\ \omega_{y,0} \equiv \dot{\theta}_0 = 194.40 \text{ s}^{-1} \\ \omega_{z,0} \equiv \dot{\phi}_0 = 267.10 \text{ s}^{-1} \end{bmatrix} \quad (23)$$

$$\omega_0 = 332 \text{ s}^{-1} \equiv 3169 \text{ rpm} \quad (24)$$

The compiled program in the area of the MatLab package obtains the laws of motion, the laws of velocity, and the laws of accelerations for all generalized coordinates of the three types of service strokes. From the laws of motion, the program obtains the trajectories.

This study shows the projections of the trajectories of the three service strokes on the coordinate planes:

The longitudinal vertical plane Oxz ;

The horizontal plane Oxy ;

The cross vertical plane Oyz .

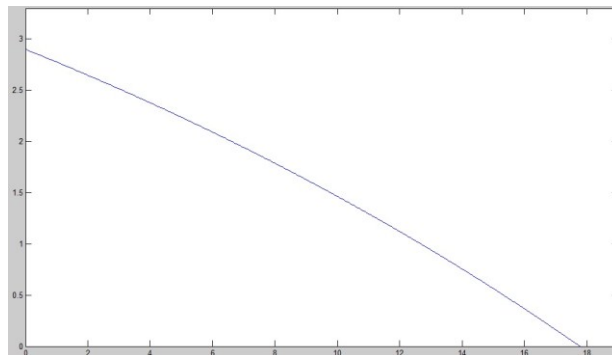


Figure 3. Trajectory of flat service stroke, projecting in longitudinal vertical coordinate plane Oxz .

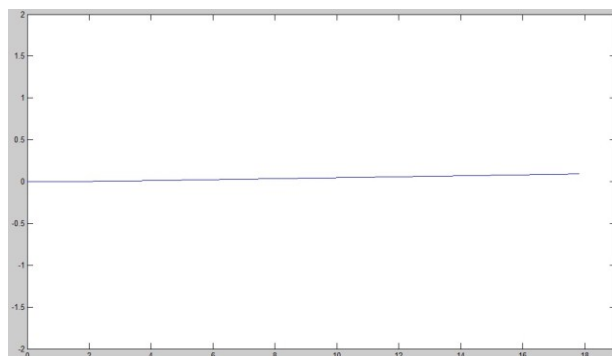


Figure 4. Trajectory of flat service stroke, projecting in horizontal coordinate plane Oxy .

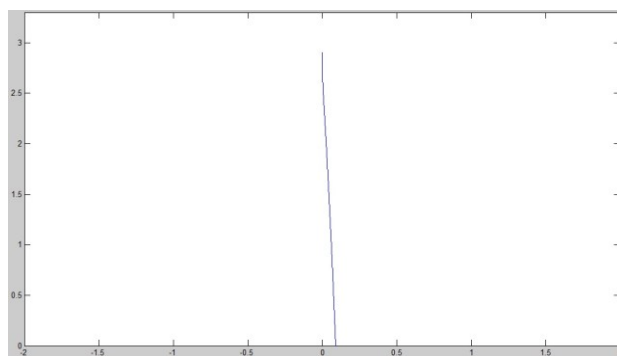


Figure 5. Trajectory of flat service stroke, projecting in cross vertical coordinate plane Oyz .

Results:

- The maximum length of the tennis ball trajectory at the moment of its contact with the court: $x_0 = 17.829 \text{ m}$;
- The Maximum transverse deviation of the tennis ball trajectory at the moment of its contact with the court: $y_0 = 0.092 \text{ m}$;
- The coordinates of the tennis ball trajectory when the ball flies over the net: $x_0 = 11.885 \text{ m}$, $z_0 = 1.139 \text{ m}$.

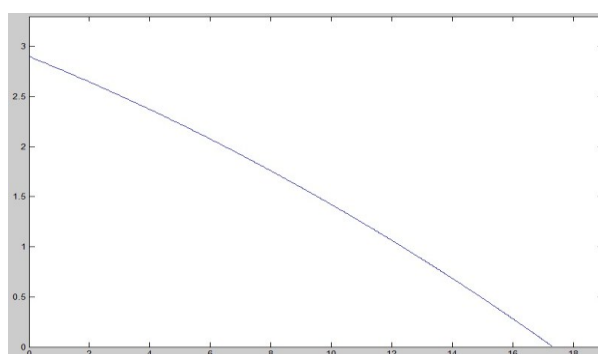


Figure 6. Trajectory of slice service stroke, projecting in longitudinal vertical coordinate plane Oxz .

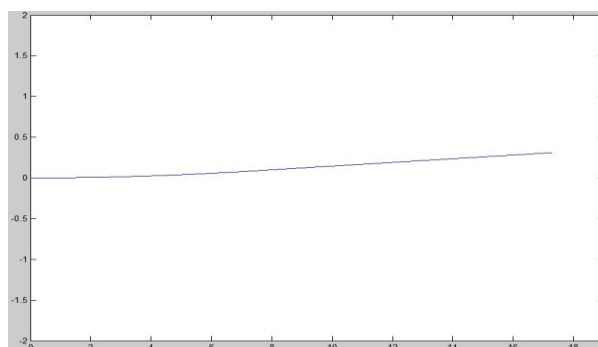


Figure 7. Trajectory of slice service stroke, projecting in horizontal coordinate plane Oxy .

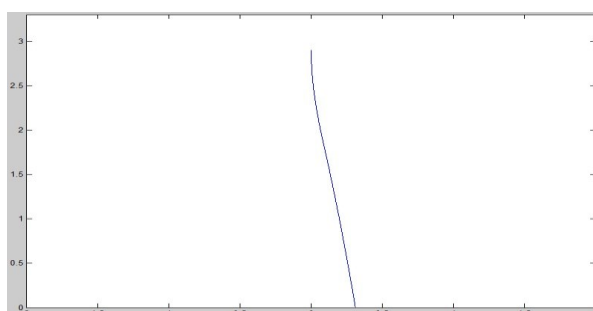


Figure 8. Trajectory of slice service stroke, projecting in cross vertical coordinate plane Oyz .

Results:

- The maximum length of the tennis ball trajectory at the moment of its contact with the court: $x_0 = 17.316 \text{ m}$;
- The Maximum transverse deviation of the tennis ball trajectory at the moment of its contact with the court: $y_0 = 0.311 \text{ m}$;
- The coordinates of the tennis ball trajectory when the ball flies over the net: $x_0 = 11.885 \text{ m}$, $z_0 = 1.084 \text{ m}$.

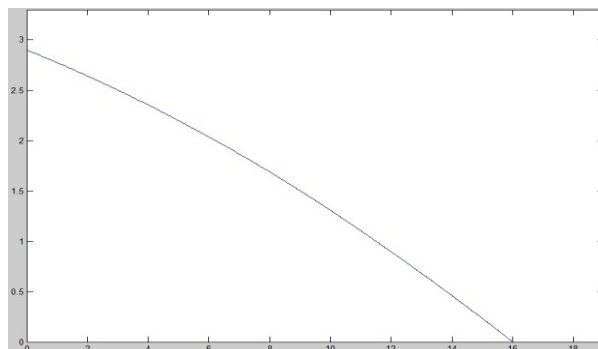


Figure 9. Trajectory of kick service stroke, projecting in longitudinal vertical coordinate plane Oxz .

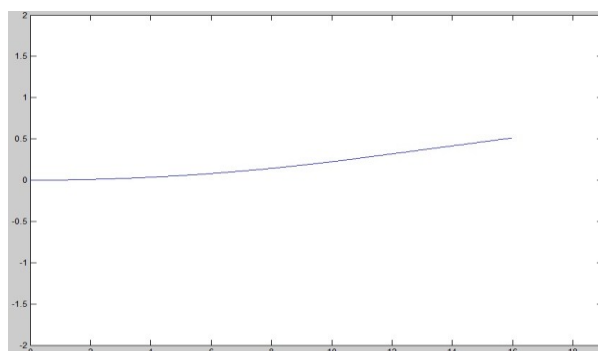


Figure 10. Trajectory of kick service stroke, projecting in horizontal coordinate plane Oxy .

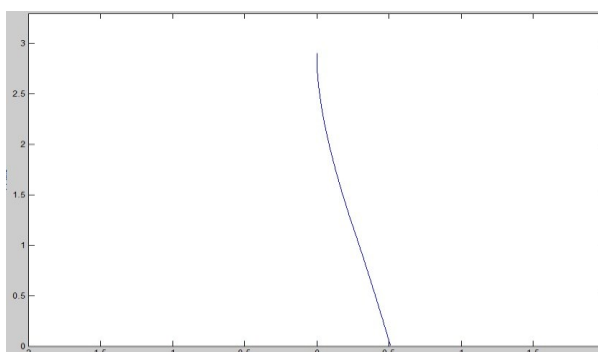


Figure 11. Trajectory of kick service stroke, projecting in cross vertical coordinate plane Oyz .

Results:

- The maximum length of the tennis ball trajectory at the moment of its contact with the court: $x_0 = 15.938 \text{ m}$;
- The Maximum transverse deviation of the tennis ball trajectory at the moment of its contact with the court: $y_0 = 0.511 \text{ m}$;
- The coordinates of the tennis ball trajectory when the ball flies over the net: $x_0 = 11.885 \text{ m}$, $z_0 = 0.917 \text{ m}$.

5. Results and Discussion

The first criterion of whether a service stroke is successful or not is whether the length of its trajectory is less than the distance to the out line of the service fields, which is theoretically in the range of 18.285 *m* to 20.052 *m*.

It can be seen that the trajectory lengths of the three types of strokes, flat, slice and kick serves, $x_0 = 17.829 \text{ m}$, $x_0 = 17.316 \text{ m}$, and $x_0 = 15.938 \text{ m}$, are smaller than 18.285 *m*.

Therefore, by this criterion, with the initial conditions, which are chosen, the three types of strokes are successful.

The second criterion is whether the tennis ball will hit the net or not.

The height of the tennis net varies from 0.914 *m* to 1.012 *m*. To these heights must be added the radius of the tennis ball. Then, this range becomes from 0.944 *m* to 1.042 *m*.

It can be seen that the heights of the trajectory of the two types of strokes, flat and slice serves, $z_0 = 1.139 \text{ m}$ and $z_0 = 1.084 \text{ m}$, are greater than the maximum height 1.042 *m*.

However, the height of the kick serve trajectory, $z_0 = 0.917 \text{ m}$, is less than even the minimum height, namely 0.944 *m*.

Therefore, according to the second criterion, with the magnitude and direction of the linear velocity of the tennis ball, which is chosen, the flat and slice service strokes are successful, while the third shot, the kick service stroke, is unsuccessful.

For the kick serve to be successful as well, a change in the direction of the initial velocity is necessary, which means a smaller angle to the horizon.

6. Conclusion

Service strokes are one of the most important shots in tennis sports. They are performed first at the beginning of each game and determine vastly the outcome of the competition. Their trajectories are primarily dependent on the initial conditions: initial position and initial linear and angular velocity. Each of these three types of strokes, flat, slice, and kick service, is characterized not only by a specific angular velocity but also by a corresponding magnitude and direction of the linear velocity. In this research, the same magnitude and direction of the linear velocity were assumed, varying with the magnitude and direction of the angular velocity. This was done in order to trace the influence of the angular velocity on the distortion of the trajectory of the tennis ball, which is due to the Magnus effect. In future research, it is necessary to trace the influence of linear velocity at a fixed angular velocity. This study will be useful not only for specialists dealing in the field of Theoretical Mechanics and Fluid Mechanics but also for tennis players and coaches.

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