AUPET Press
Technical Journal of Daukeyev University
Volume 2, Issue 2, 2022, pp. 66-72
ISSN 2788-5488

Approximate natural frequencies of AFGM beam

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Received 05 September 2022; Accepted 27 September 2022; Published 13 October 2022
DOI: https://doi.org/10.52542/tjdu.2.2.66-72

Abstract. This article gives the approximate natural frequencies of axial functionally graded material (AFGM) beams. Based on Matlab software and the traditional finite element method, we can estimate the natural frequencies for this structure under four types of boundary conditions. Some examples are given which prove that this way is simple and applicable.

Keywords: approximate, frequencies, software, strength, beam, Matlab.

1. Introduction

Another composite material called functionally graded material (FGM) has been widely used in the last few years due to its distinct features. FGMs are materials that consist of a combination of two different separate components with continuous gradient mechanical and physical characteristics. Normally, these features are normally distributed along particular directions. Generally, FGMs are composed of metal and ceramic; metal prevents material from breaking when subjected to heat stress while the ceramic, with high thermal resistance-low strength, offers resistance at extreme temperatures. In addition, these materials can be characterized by toughness, high strength and resistance to high temperature and corrosion. In conventional composite materials, the interfacial stresses occur at the interfaces between two different constituent materials due to differences in the material properties. FGMs overcome this problem making composites materials can be applied to vehicles, airplanes, military projects, and biomedical field and so on. Due to its advantages, several articles have been accomplished on using FGMs for several applications in order to understand its behavior under different working conditions. In reference [1], the generalized differential quadrature method was utilized to study the modal characteristics of clamped-clamped AFGM beam. The material properties were assumed to vary in axial direction in accordance with power law distribution. The Hamilton's principle was used to derive the governing equations. The available partial differential equations were converted into linear algebraic form using generalized differential quadrature method. By the same authors, the harmonic differential quadrature method was employed to investigate the vibration characteristics of axially functionally graded tapered beam. The influence of non-homogeneity parameter, taper ratio and aspect ratio on the frequencies of the AFGM tapered beam was discussed in this reference, [2]. In [3], the vibration problem of beams with axial functionally graded materials and variable thickness was firstly investigated by isogeometric analysis in conjunction with three-dimensional theory. Based on guaranteeing geometry exactness of the strength of non-uniform rational B-splines, the curves of non-uniform thicknesses of beams were exactly described. Two beam models (slender model and plump model) were taken into account in this research. The article [4] studied the free vibration of an axially functionally graded beam spinning with constant angular speed about its longitudinal axis. Its whirling frequencies, critical speeds, and mode shapes were obtained in order to investigate the influences of axially functionally graded material. Spinning Timoshenko beam model was employed to incorporate the effects of transverse shear, rotational inertia, and gyroscopic motion. A spectral-Tchebychev method was also employed to derive the discrete governing equation and to solve the dynamic characteristics of the beam. The static deflection of axially functionally graded cantilever beam was computed using Rayleigh and Finite Element methods considering Beam Theory of Euler-Bernoulli. In Rayleigh method, the equivalent stiffness of two kinds of cantilever

FG beams were calculated for different power law index and then used to calculate the static deflection. The Finite Element model was built by ANSYS APDL version 17.2 using BEAM189 element. The effects of number of segment, power law index and type of applied load on the dimensionless deflection were studied, [5]. Furthermore, in literature [6], non-uniform beam with lengthwise material variation resting on Pasternak foundation was investigated for free vibration. A beam with its material properties and thickness varying linearly along its length was selected for investigation. Euler-Bernoulli beam theory along with Rayleigh-Ritz method was employed for the formulation and the equation of motion was derived using Hamilton's principle, etc. This article gives the approximate natural frequencies of AFGM beams based on Matlab software and the traditional finite element method with the simple element model. This article is divided into four parts. Following part 1 as above, we continue to present the formulations of the axial functionally graded material for beam in part 2 as well as show some essential results in part 3. Finally, a comment is also given in part 4.

2. Axial functionally graded material for beam

Consider an AFGM beam with length L, as shown in Figure 1, whose cross-section is rectangular and the depth and breadth are h and b. In this work, the beam is composite of *ceramic* and *metal* and they are varying from the right side (*ceramic*) to the left side (*metal*) of the beam. The volume fraction of the ceramic constituent V_L for AFGM beam i:

$$V_L = \left(1 - \frac{x}{L}\right)^P \tag{1}$$

$$V_L + V_R = 1 \tag{2}$$

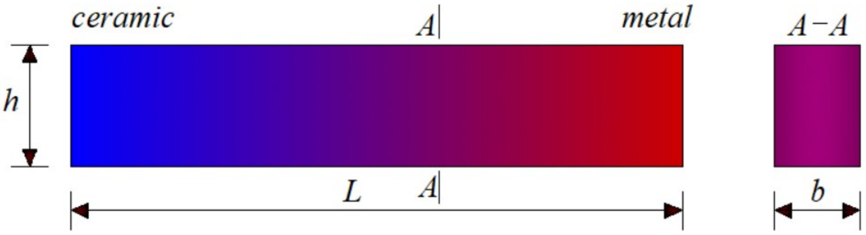


Figure 1. An AFGM beam.

in which V_L and V_R are the volume fractions of the constituent at left and right end, respectively, p is the nonnegative variable parameter (or gradient index) which dictates the material variation profile throughout beam length. Different material distribution can be achieved just by changing the gradient index, then different effective material properties are demonstrated. The effective properties of the AFGM can be given using the rule of mixture method as:

$$P(x) = P_L V_L + P_R V_R \tag{3}$$

$$P(x) = (P_L - P_R)\left(1 - \frac{x}{L}\right)^P + P_R \tag{4}$$

where P_L, P_R are the effective material properties of the AFGM beam at the left and the right end of the beam, respectively. It is important to mention that the effective material properties such as Young's modulus (E), Poisson's ratio (ν) and mass density (ρ) vary continuously in the longitudinal direction. Figure 2 shows the variation of the volume fraction of the ceramic constituent along the length of beam.

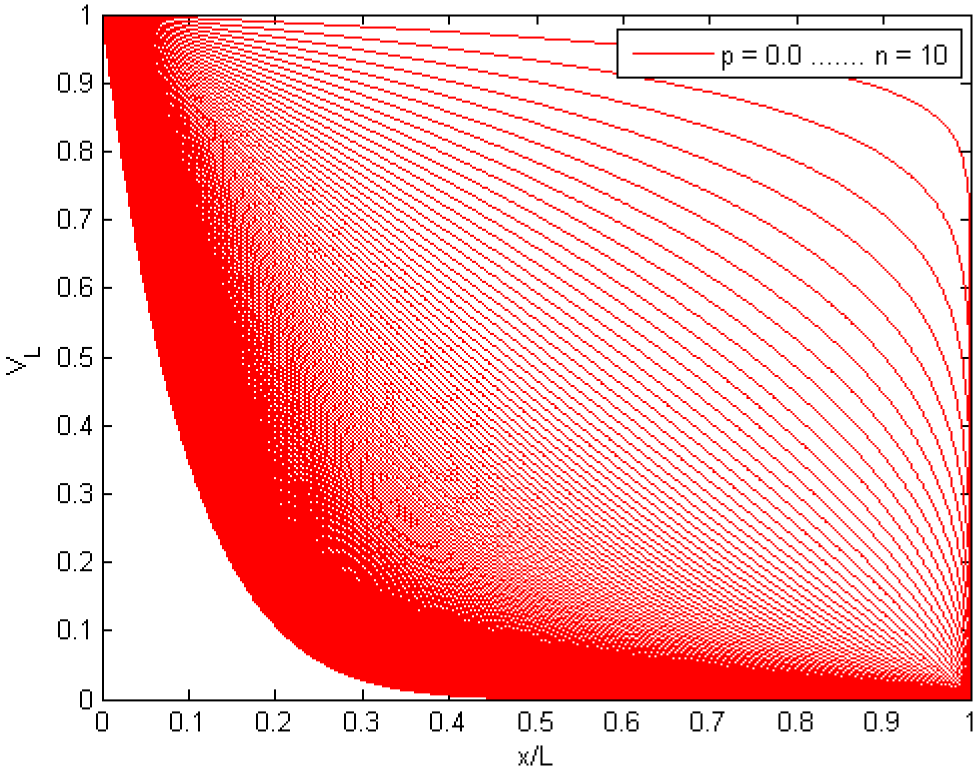


Figure 2. The volume fraction V_L.

Based on the simple beam element [7], the free vibration results can be obtained in the next part.

3. Results And Discussions

In this section, as a first example, the validity is checked for an isotropic cantilever beam with length $L = 0.5$ m, width $b = 0.05$ m and height $h = 0.02$ m by following [8]. The material properties are: Young's modulus $E = 2.1 \times 10^{11}$ Pa, the density $\rho = 7800$ kg/m^3. The proposed results for the first three natural frequencies are compared with analytical solutions [8] as shown in Table 1. The analytical solutions display as:

$$f_1 = \sqrt{\frac{EI}{A\rho}} \frac{1}{2\pi} \left(\frac{k_1}{L} \right)^2 ; \quad k_1 = 1.875 \tag{5}$$

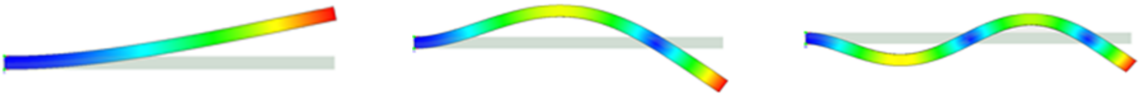
$$f_2 = \sqrt{\frac{EI}{A\rho}} \frac{1}{2\pi} \left(\frac{k_2}{L} \right)^2 ; \quad k_2 = 4.694 \tag{6}$$

$$f_3 = \sqrt{\frac{EI}{A\rho}} \frac{1}{2\pi} \left(\frac{k_3}{L} \right)^2 ; \quad k_3 = 7.855 \tag{7}$$

Besides, the first three mode shapes [8-9] can also be seen in Figure 3.

Table 1. Verification of the first three natural frequencies.

Mode	Article	Analytical
1	67.05	67.0
2	420.23	420.2
3	1176.70	1176.7



Mode 1 Mode 2 Mode 3

Figure 3. The first three mode shapes of cantilever beam.

Next, the material of an AFGM beam is assumed to be material in which the right side is steel with E_R = 209 GPa and ρ_R = 7800 kg/m^3 while the left side is alumina Al_2O_3 with E_L = 390 GPa and ρ_L = 3960 kg/m^3. The parameters of cross-section and length are the same as in the above example. The first five natural frequencies will be calculated and evaluated through the change of n as well as the boundary conditions. In Table 2, with four values of n and four types of boundary condition: CF, CC, CS & SS, the article shows the first five approximate natural frequencies. The values of these frequencies decrease with the increasing value of n for all types of boundary conditions, respectively. Figures 4 and 5 further illustrate this comment.

Table 2. The first five natural frequencies of AFGM beam by changing boundary conditions and n.

CF	***n***			
	1	5	20	100
# 1	91.2	77.6	70.6	67.7
# 2	572	460.8	436.9	424.1
# 3	1604.2	1289.6	1214.8	1186.9
# 4	3145.6	2531.2	2371	2324.4
# 5	5201.6	4189.8	3911.9	3840.4
CC	***n***			
	1	5	20	100
# 1	576.3	468.2	443.8	430.7
# 2	1597.3	1289.1	1214.2	1186.4
# 3	3138.8	2531.2	2371.1	2324.5
# 4	5194.5	4189.8	3911.9	3840.4
# 5	7764.5	6265.1	5838.4	5734
CS	***n***			
	1	5	20	100
# 1	398.3	323.5	306.5	296.8
# 2	1295.8	1044	985.2	961.3
# 3	2708.6	2181.7	2045.9	2004.5
# 4	4635.9	3736.1	3490.5	3426
# 5	7077.4	5707.2	5320.5	5225.2
SS	***n***			
	1	5	20	100
# 1	256.2	197	188.2	187.8
# 2	1027.2	815.7	756.6	751.2
# 3	2312.3	1853	1711.3	1690.4
# 4	4111.6	3307	3054.9	3005.6
# 5	6424.9	5177.2	4787.3	4697.1

Finally, by changing the ratio L/h from 25 to 5, a SS-AFGM beam with length L = 0.5 m, b = 0.05 m and n = 5 is modeled and estimated the first five natural frequencies. Table 3 and Figure 6 show that all values of these frequencies increase with the decreasing ratio L/h.

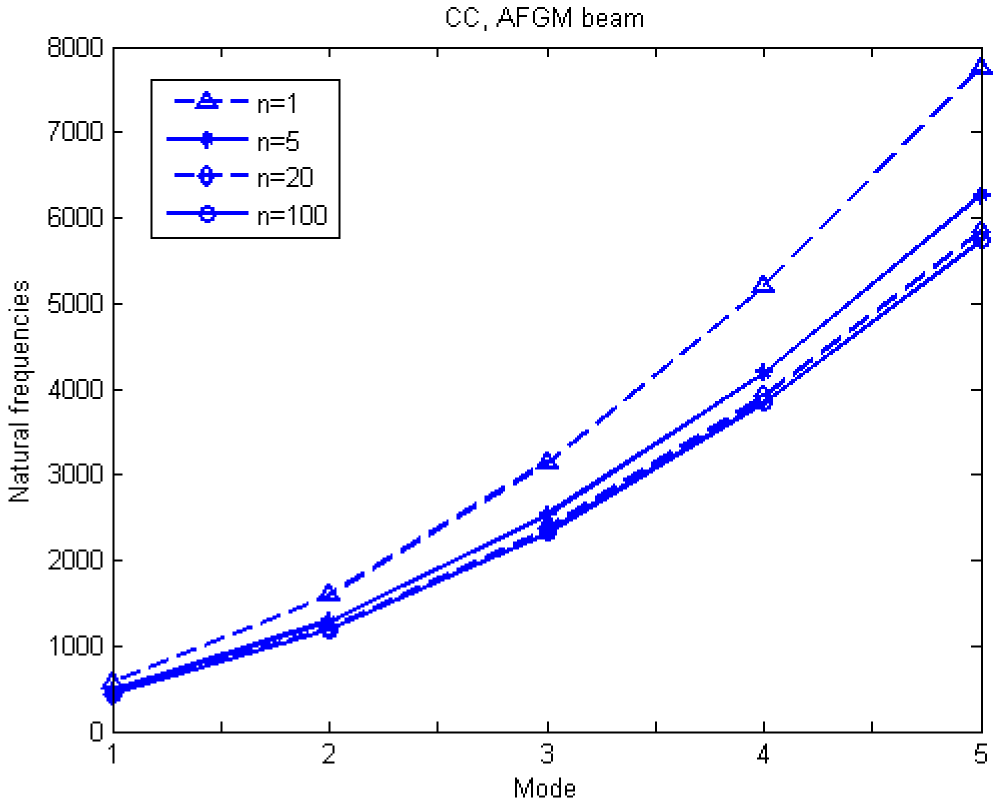


Figure 4. The first five natural frequencies of CC-AFGM beam by changing *n*.

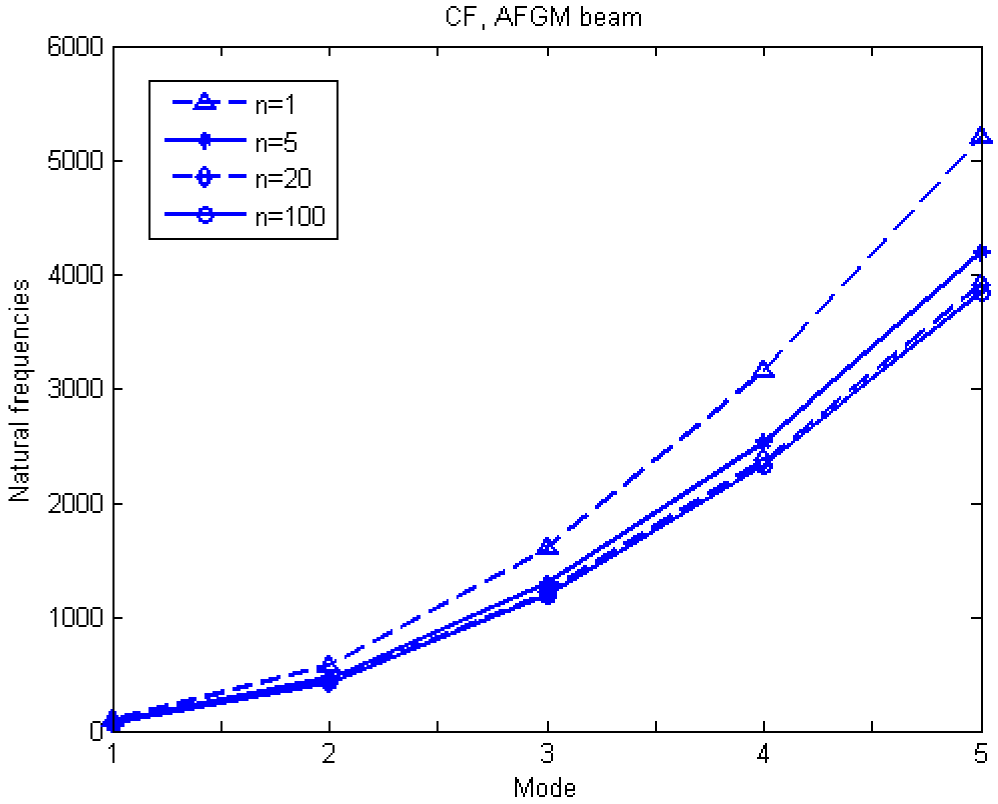


Figure 5. The first five natural frequencies of CF-AFGM beam by changing *n*.

Table 3. The first five natural frequencies of SS-AFGM beam by changing *L/h*.

SS *n* = 5	***L/h***				
	25	20	15	10	5
# 1	197	246.2	328.3	492	985
# 2	815.7	1019.6	1359.5	2039	4079
# 3	1853	2316.3	3088.4	4633	9265
# 4	3307	4133.7	5511.6	8267	16535
# 5	5177.2	6471.5	8628.6	12943	25886

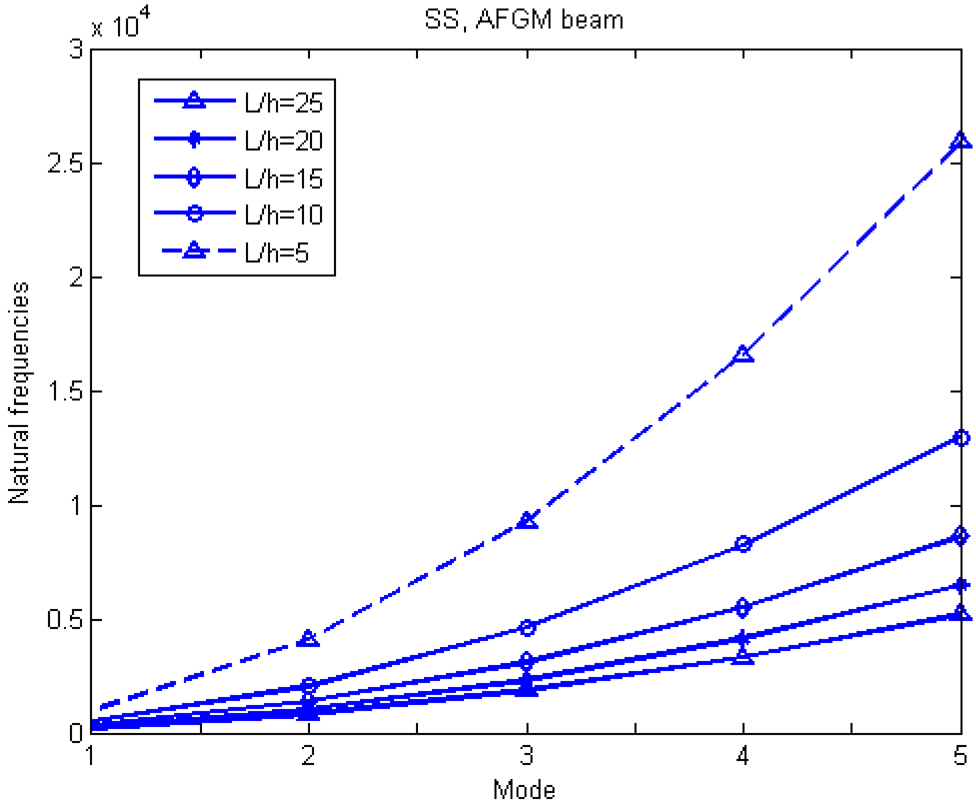


Figure 6. The first five natural frequencies of SS-AFGM beam by changing *L/h*.

4. Conclusion

This article shows the approximate natural frequencies of AFGM beams. Based on Matlab software and FEM with a simple beam model, we can estimate these frequencies for AFGM beams under the influences of boundary conditions as well as the ratio *L/h*. Some examples of estimating natural frequencies for AFGM beams are given, which prove that this way is simple and applicable.

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